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APPLETONS' STANDARD ARITHMETICS

NUMBERS APPLIED

NEW YORK
PUBLIC
LIBRARY

A COMPLETE ARITHMETIC

BY

ANDREW J. RICKOFF, A.M., LL.D.

PART I

NEW YORK .:. CINCINNATI .:. CHICAGO
AMERICAN BOOK COMPANY
FROM THE PRESS OF
D. APPLETON & COMPANY

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PUBLIC LIBRARY
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PREFACE TO PART I.

THIS work is not the result of any ambition on the part of the publishers to add another title to their already long list of text-books, but of a desire to meet a wide-spread and growing demand for a treatise on arithmetic adapted to the objective methods of instruction now so common in all educational institutions which have been reached directly or indirectly by the influence of normal schools, teachers' institutes, etc.

In its preparation the author has kept steadily in view these two thoughts: (1) That words are useless in the ratio that they fail to call up in the mind vivid images of the things signified. Hence the aim to vitalize the relation of words and things by the aid of the best practicable illustrations at every point; and (2), That, to the learner, the processes of arithmetic are apt to be but blind manipulations of figures after prescribed models, unless he realizes the fact that they are only convenient substitutes for operations that may be applied to the objects which they represent.

The book is intended to be put into the hands of the learner as soon as he has completed a course in primary arithmetic; but it would be well for him to begin the study of it with the first chapter, that he may get a better technical knowledge of the fundamental rules and their relations to each other, and that he may become rapid and reliable in computations involving integers before he takes up the more complicated subject of fractions.

Great care has been taken to adapt the work as far as possible to the needs of the great number of children who are withdrawn from school before a full course in arithmetic can be completed. With this object in view, the more useful business applications of elementary principles are made as soon as they are learned. Thus, familiar measures are introduced before reduction is mentioned; federal money before decimals, etc.

The knowledge of percentage and interest to be gained by the study of the pages devoted to these subjects, in advance of their more formal treatment in Part II, will be found amply sufficient to meet the demands of all common business affairs.

18972

Attention is respectfully called

1. **To the simple treatment** of the decimal system of notation, and the great number of exercises intended to familiarize the pupil with the facilities for calculation which it affords.

2. **To the multiplicity of short exercises** that can be performed without the aid of the pencil, or that require but few figures in their solution. Longer ones are not wanting to test the perseverance of the pupil.

3. **To the directions to the pupil, having in view the formation of right habits of computation.** The "making up" method of subtraction, and the so-called "continental" method in division, though not obtrusively presented, are worthy of the attention of teachers. The latter furnishes an excellent mental exercise.

4. **To the suggestions for original problems**, now so commonly resorted to by the best teachers to stimulate the interest of their pupils, and to give them a better understanding of the subjects to which they relate. Their usefulness as short practical exercises in penmanship, spelling, and composition, will be appreciated by all.

5. **To the simple and direct methods** of treating the fundamental rules, common and decimal fractions, percentage and interest.

6. **To the rigorous adherence throughout the work to the inductive methods of instruction.**

7. **To the great number of purely mental exercises** to be found in this book. Closely associated as these are with the corresponding steps in what is commonly called written arithmetic, they aid the student in acquiring a thorough understanding of the particular subject in hand, as well as facility in computation without the aid of the pencil.

The number and variety of exercises and problems in this work are so great as to supersede any necessity for supplementary books in either mental or written arithmetic.

NEW YORK CITY, *May 14, 1887*

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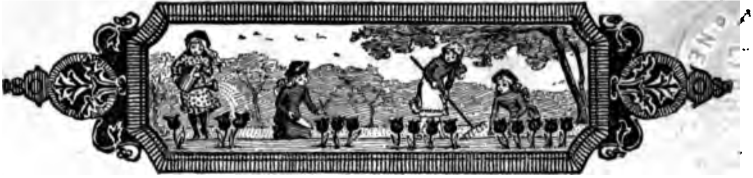


APPLETONS' Standard Arithmetic.

CHAPTER I.

NOTATION AND NUMERATION.

The Writing and Reading of Numbers.

OBJECTS.	NUMBER.		OBJECTS.	NUMBER.	
●	*One*	1	●●●●●●	*Six*	6
●●	*Two*	2	●●●●●●●	*Seven*	7
●●●	*Three*	3	●●●●●●●●	*Eight*	8
●●●●	*Four*	4	●●●●●●●●●	*Nine*	9
●●●●●	*Five*	5	●●●●●●●●●●	*Ten*	..

1. The signs 1, 2, 3, 4, 5, 6, 7, 8, 9, are called the ***nine digits***; because first used to represent a number of fingers.

The word *digit* is sometimes used for the word *finger*.

The following are the written forms of the digits:

1, 2, 3, 4, 5, 6, 7, 8, 9.

Tens and Units.

2. When we count more than nine we begin to count by tens and ones. After *nine* we say *ten,* then *eleven,* which means ten and one, *twelve* (ten and two), *thirteen* (ten and three), *fourteen* (ten and four), etc., to *nineteen* (ten and nine). Then we come to *twenty,* which means two tens, *twenty-one* (two tens and one), etc., after twenty-nine we have thirty, forty, fifty, etc.

Counting.—Count the balls of the numeral frame or other objects from ten to ninety-nine, as follows:

ten and one	**two tens and one**	**three tens and one**
ten and two	**two tens and two**	**three tens and two**
ten and three	**two tens and three**	**three tens and three**
etc., to	**etc., to**	**etc., to**
ten and nine	**two tens and nine**	**three tens and nine**
two tens	**three tens**	**four tens, etc.**

Writing.—We may write these numbers by using the digits 1, 2, 3, etc., instead of the words one, two, three; thus:

1 ten and 1	**2 tens and 1**	**3 tens and 1**
1 ten and 2	**2 tens and 2**	**3 tens and 2**
1 ten and 3	**2 tens and 3**	**3 tens and 3**
etc., to	**etc., to**	**etc., to**
1 ten and 9	**2 tens and 9**	**3 tens and 9**
2 tens	**3 tens**	**4 tens**

and so on to 9 *tens and* 9.

3. But the writing of numbers is still further shortened by omitting the words *ten and,* or *tens and;* thus, for *1 ten and 1* we write 11; for *1 ten and 2* we write 12; for *2 tens and 1* we write 21, and so on up to 99 (9 tens and 9). Thus we can tell whether a digit stands for *ones* or tens by the place it occupies. If it represents *ones,* it has the first place at the right; if *tens,* it occupies the next on the left.

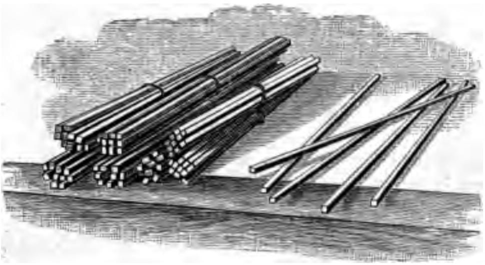
4. The right-hand place is called the ***units'*** or ***ones' place.*** The next place to the left of the units is the ***tens'*** place.

Note.—We use the word *unit* for the word *one*, and *units* for *ones*. See Art. 12.

If, as in 1 ten, 2 tens, etc., there are no *ones* or *units* to be expressed, we write the figure 0 in the *units'* place; thus, 10, 20.

5. The figure 0 does not express any number, but it is used to fill vacant places, as in the case above. It is called ***cipher*** or ***zero.*** It is a figure, but not a digit.

6. This is the ***decimal*** or ***tens' system*** of counting and of writing numbers.



Note.—For a simple illustration of this system, imagine a boy counting sticks. He counts ten and ties them together; then ten more, and so on till he has seven bundles and five sticks over. Then, writing 75, he shows that he has seventy-five sticks, the 7 being in *tens*, and the 5 in *units'* place.

7. Between twelve and twenty we call the *ten "teen,"* which means *"and ten";* as, *fourteen,* that is *four* and *ten.* From nineteen to ninety-nine we call the *ten "ty,"* which means *"times ten";* as, *sixty,* or *six times ten; seventy-five,* or *seven times ten and five units.*

EXERCISES IN WRITING AND READING NUMBERS.

Note.—Pupils should follow the forms of the digits given at the top of the preceding page, or other good copy. Let him here lay the foundation of neatness and accuracy in the writing and use of figures.

1. Write very neatly, in figures, the numbers from one to nine; from thirty to thirty-nine; forty to forty-nine; nineteen to ten; sixty to sixty-nine; ninety to ninety-nine; eighty-nine to seventy; fifty-nine to fifty; thirty-four to fifty-six.

2. Write in words: 75, 43, 51, 98, 29, 83, 11, 32, 64, 49, 17, 56, 19, 68, 31, 77, 99, 10, 24, 48.

Note.—The pupils may be required to show a number of jack-straws or of other objects equal to the numbers expressed. They should be arranged appropriately in tens and units.

3. How many tens are in twenty-eight? In sixty-two, fifty-six, ninety-five, eighty, seventy-one, forty-eight, sixty-five, thirty-three, fifty-one, etc.?

4. Write the following numbers in a column, and opposite each one express the same value in words: 92, 65, 38, 71, 40, 83, 14, 54, 17, 26, 90, 12, 83, 24, 75, 36.

5. The number 39 is expressed by two digits. What does the 9 stand for? The three? Which one, as it stands here, expresses the greater value? Why?

6. Answer the same questions in regard to 89, 48, 56, 35, 67, 22. Illustrate by objects.

Note.—In English the digit expressing the tens is generally read first; as, forty-eight; but in the German language they say: eight and forty. Sometimes we hear the same in English.

7. Read the following numbers in the German way: 27 (seven and twenty), 36, 58, 67, 89, 38, 45. If the places of these digits were exchanged, would the numbers thus expressed be larger or smaller? Why? Illustrate by objects.

8. What is the greatest number that can be expressed by two figures? What is the smallest?

9. Write in figures: Seventeen, thirteen, fifteen, twenty-eight, ninety-five, forty-two, eighty-three, thirty-four, sixty-nine, seventy-seven, eighteen, fifty-one, sixty-seven, forty-eight, ninety-five, eighty-eight, thirty, sixty.

10. Read the following. (May be copied, or written at dictation, and then read.) 10, 19, 13, 18, 12, 17, 11, 16, 14, 20, 23, 21, 25, 27, 29, 22, 24, 26, 28, 32, 36, 31, 35, 33, 38, 37, 49, 44, 48, 41, 47, 50, 56, 52, 63, 75, 84, 95, 58, 67, 78, 89, 91, 65, 85, 96, 72, 15, 30, 35, 39, 42, 55.

11. The pupil may copy the following numbers:

56 34 67 23 11 89 78 45 98 10 87 54 32 43 21
65 76 17 26 49 61 88 39 58 72 99 30 27 83 57

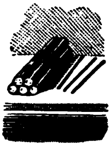
Note.—Care should be taken that each number be recognized as a whole, that the pupil may not copy figures merely. He should recognize *56* as *fifty-six*, and not as the figures 5 and 6.

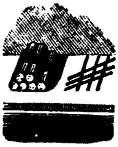
12. Take in at a glance as many numbers as you can, and repeat them, looking off the book:

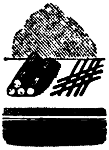
16 23 62 43 50 84 31 74 39 20
78 47 39 91 98 67 17 58 46 55

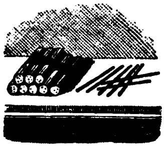
13. The teacher may dictate two or more numbers at a time from exercises 10 and 11.

14. Write the number of jack-straws represented in each group below. The bundles are of ten each.









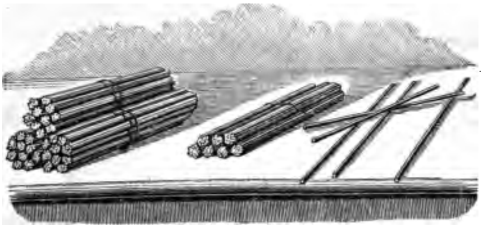
Suggestion.—Let the pupils make original notation exercises similar to the above, arranging the objects in groups, and noting the number both in words and figures.

8. Hundreds.—If we count one more than ninety-nine we shall have nine tens and ten ones, or *ten tens.* Ten tens make *one hundred.* To express one hundred in figures we write *1* in the third place, thus, *100,* filling the places of tens and units with ciphers. The *1* now stands for *one hundred.* A digit in the third place from the right stands for *hundreds,* and hence we write:

100 (one hundred),
200 (two hundred),
300 (three hundred),
400 (four hundred),
500 (five hundred),
600 (six hundred),
700 (seven hundred),
800 (eight hundred),
900 (nine hundred).

If with the hundreds we have to write any number of tens, as three hundreds *and* seven tens, we place the digit representing the tens in the tens' place; thus, 370, read (3 hundreds 7 tens), three hundred seventy.

Again, if with the hundreds we have to write any number of ones, as three hundreds and five ones, we place the figure representing the ones in the ones' place; thus, 305.



Three hundreds, seven tens and five ones here represented are written thus: 375, and read *three hundred seventy-five.*

We have learned, 1st, that *ten ones* make *one ten,* and *ten tens* make *one hundred;* 2d, that in writing numbers the place on the right is the ***ones' place;*** the next, the ***tens' place;*** and the next, the ***hundreds' place.***

EXERCISES IN WRITING AND READING NUMBERS.

1. Express in figures: One hundred, six hundred, nine hundred, seven hundred, four hundred, two hundred, etc.

2. Also, one hundred thirty, six hundred twenty, five hundred eighty, three hundred fifty, two hundred seventy, etc.

3. Also, one hundred sixty-five, three hundred eighty-four, nine hundred seventy-one, four hundred thirty-three, etc.

4. How many hundreds in 481? How many tens? How many ones? How many of each in 385, 610, 974, 572, 137, 448?

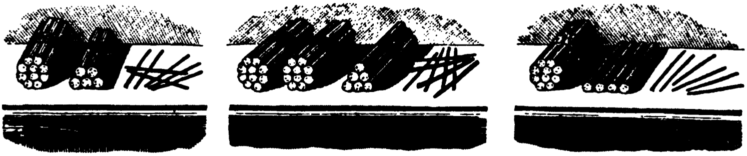
5. Write the following numbers in a column, and opposite each the same number in words: 218, 117, 916, 675, 854, 370, 523, 388, 446, 770, 978, 101, 340, 620, 304.

6. Read, taking in at one glance as many numbers as possible:

100	201	310	404	500	691	700	800	909
102	204	320	440	572	673	719	808	910

Note.—The foregoing numbers may be read in lines or columns, forward or backward, as the teacher may direct.

7. Write in figures the number of jack-straws represented in each of these groups.



8. Read 792. What does the 2 stand for? The 9? The 7?—Show what each figure stands for in the following numbers: 439, 562, 101, 760, 875, 460, 140, 104, 583, 61.

9. In a number of three places, which figure is read first? Which represents the highest order? How would you write three hundred nine, having no tens? How would you write 7 hundred twenty, having no ones? Will it do to leave the place of the ones or tens vacant? Why?

10. What is the largest number that can be represented by three figures? What is the smallest whole number?

11. Write in figures: Three hundred fifty, six hundred eighty, two hundred seventy, eight hundred fifteen, four hundred twenty-eight, nine hundred nine, one hundred ninety-six.

12. Copy the following, glancing at each number but once: (Think of the numbers represented, not merely of the figures to be written.)

107 400 212 560 309 653 356 365 635 536
801 118 180 870 357 429 560 608 742 897
215 419 711 999 233 100 677 822 301 405
103 205 340 409 583 655 728 846 979 893

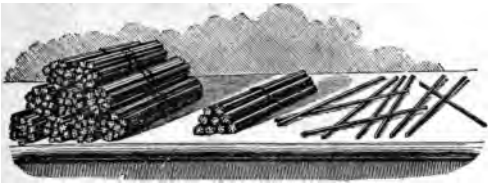
13. **Note.**—The teacher may also dictate two or more of the foregoing numbers at once, thus quickening the attention of the pupils.

14. Write in regular order the numbers from 150 to 199; from 260 to 299; from 307 to 328; from 480 to 499; from 585 to 602; from 687 to 706; from 791 to 809.

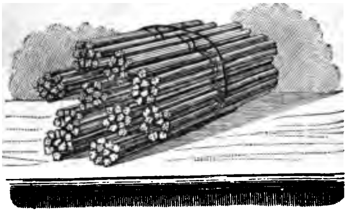
15. Write in words the numbers from 337 to 345; also from

883 to 890; from 555 to 563; from 98 to 104; from 872 to 883; from 190 to 205.

9. Thousands.—The greatest number we have written thus far is 999, or 9 hundreds, 9 tens, 9 ones. If we count one more, the nine ones at the right hand will become *one ten;* and putting this with the nine tens, we have *ten tens,* or *one hundred.*



Putting this one hundred with the nine hundreds, we have *ten hundreds;* and, as we made *one ten* out of ten ones, and *one hundred* out of ten tens, so we make *one thousand* out of ten hundreds. Thus, after adding one stick to the nine hundred and ninety-nine shown on the table above, the result would be as represented in this picture.



To express one thousand in figures, we write *1* in the fourth place; thus, *1000,* filling the places of hundreds, tens, and units with 0's. The *1* now stands for one *thousand.* A digit in the fourth place stands for thousands, hence we have 2000 (two thousand), etc. Hundreds, tens, and units, if any, fill their proper places.

EXERCISES IN READING AND WRITING NUMBERS.

1. Read 77, 15, 93, 106, 601, 810, 7080, 9107, 5006, 561, 3091.

2. *Write at dictation and read:*

5783	2100	9009	1706	5430	8071	2360	3902	1003	5701
6702	4000	3201	4300	5701	7010	8090	9100	1901	7707

3. Write ten such numbers as you please, and read them.

4. In the following numbers, how many units, tens, and hundreds are expressed by the figures in those orders?

2375	1318	2380	3689	7401	8120	9000	7895
3624	5074	6138	8376	2380	8016	7980	1234

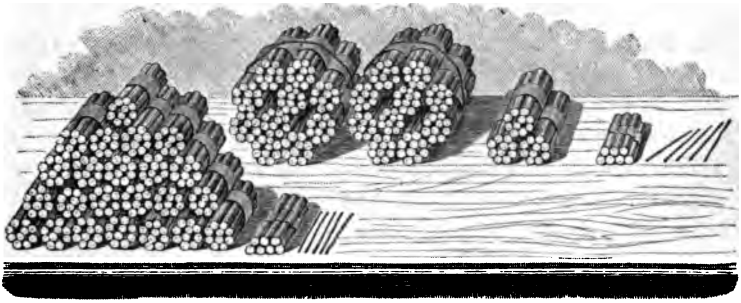
5. What is the smallest whole number that can be expressed by four figures? What is the greatest?

6. *Read:*

6789	9867	5432	4756	8912	3129	7891	4321
6000	6600	6660	6666	7654	6035	8765	8003
1098	2076	3054	4032	5010	7123	7009	5273
9002	9387	8793	1100	4002	7628	9347	6102

7. Read the foregoing columns downward and upward, and the lines from right to left and left to-right. They may also be written at dictation and read.

8. Read the numbers of Exercise 4, reading the thousands and hundreds together as hundreds. Thus, 2375=twenty-three hundred seventy-five.



9. Copy the numbers of Exercise 6. No copying of single figures should be allowed; the number should be recognized and written as a whole.

10. Which figure in a number of four places is read first? Which represents the highest order? Which the lowest?

11. How many ciphers are needed in 4 thousand 17? Why?

What difference is there between the written forms 468 and 4608? Between 375 and 3705?

12. Is there any difference between the numbers indicated by 46 in 468 and in 4608? Is the value of 8 in one number different from its value in the other? Why?

13. What number is expressed by the figure 9 in 7009? In 7900? In 7090?

Review Exercises.

1. Write in columns the figures which express the following numbers:

four	forty	four hundred	four thousand
one	ten	one hundred	one thousand
five	fifty	five hundred	five thousand
two	twenty	two hundred	two thousand
nine	ninety	nine hundred	nine thousand
six	sixty	six hundred	six thousand
three	thirty	three hundred	three thousand
eight	eighty	eight hundred	eight thousand

2. How can you make the digits in your first column express tens? (Answer: By annexing a cipher.) How hundreds? How can you make the digits in the second column express units? How hundreds? How can you make the digits in the third column express tens? How ones? (Answer: By erasing two ciphers.) Make the digits of the fourth column express hundreds; also tens. Will it change the value of the digits to place a cipher at their left?

3. *Express in figures:*

sixteen
twenty-nine
fifty-two
thirty-six
seventy-eight
forty-five
ninety-four
eighty-three

one hun. seven
three hun. eighteen
five hun. twenty-six
seven hun. sixty-four
eight hun. fifty-six
nine hun. thirty-eight
two hun. eighty-nine
four hun. forty-five

three thou. seven hun. eight
six thou. one hun. twelve
one thou. six hun. thirteen
eight thou. two hun. twenty
three thou. four hun. fourteen
four thou. nine hun. ninety
two thou. six hun. ten
five thou. three hun. thirty

4. Make 60, 40, 80, 10, 30, 50, 90, 70, 20 larger by 100; by 300; by 500; by 400.

5. Make 61, 42, 53, 74, 85, 26, 37, 18 larger by 400; by 200; by 700.

6. Would it alter the value of 8 in 81 if you were to place a cipher on the right of the 1? Answer similar questions in regard to the first figures in 29, 36, 45, 19, 58, 67, 71.

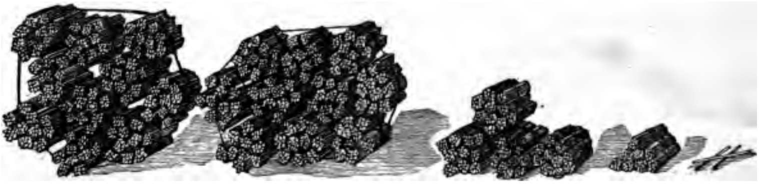
7. How many thousands, hundreds, tens, and units are expressed in each of the following numbers: 3624, 5781, 9010, 8107?

---

8. Express by figures the number of sticks represented in the 1st or units' group; also in

the 3d
the 2d
the 4th
the 4th and 3d
the 3d and 1st
the 3d and 2d
the 2d and 1st
the 4th and 1st
the 4th and 2d
the 3d, 2d, and 1st
the 4th, 3d, and 1st
the 4th, 2d, and 1st

What number is represented in all the groups together?



Note.—Pupils should prepare suitable objects for such illustrations. Bundles of tens, hundreds, etc., with single objects, should often be arranged promiscuously, and the learner be required to write the number in figures. Let him observe that we may estimate the value of the bundle by its size, but whether a digit represents tens or thousands depends on the place it occupies.

9. Write in columns, of ten each, all the numbers from one hundred to one hundred fifty-nine. Also, from two hundred fifty-one to three hundred. Also, from seven hundred eighty-three to eight hundred thirty-two. Also, from six hundred twenty-two to seven hundred one.

10. Write all the numbers from one thousand six hundred seventy-three to one thousand seven hundred two. Write in columns of ten numbers each.

10. After Thousands.—1. We have thus learned, ***First***, the names of the orders to thousands; *Second*, that ten of any order make one of the next higher; and, *Third*, that the order of a figure—that is, whether it represents units, tens, hundreds, or thousands—is known by the place which it occupies, numbered from the right.

2. When we reach a thousand we begin to count the thousands as we did the units or ones: that is, we count 1 thousand, 2 thousand, up to 999 thousand, and when we have a thousand thousand we call the number one million. Millions we count in the same way: that is, 1 million, 2 millions, etc., up to 999 millions. When we reach a thousand millions, we call the number a billion. Billions we count in the same way, so trillions, quadrillions, etc.

3. In writing these numbers we might write the number of thousands as we do the numbers up to a thousand, and attach the word thousand to each number; thus, 8 thousand, 76 thousand, 999 thousand, etc., etc. But just as we avoid writing the words units, tens and hundreds, by giving to each order its place, so we avoid writing the word thousand by giving thousands the three places to the left of hundreds. In the same manner we give millions the three places to the left of thousands.

4. In this way it comes that, when more than three figures are employed to express any whole number, they are divided into groups, the first of which, numbering from the right, is used to denote any number from 1 to 999 units; the second, from 1 to 999 thousand; the third, from 1 to 999 million, etc. These groups are called ***Periods***, and, for convenience in reading, are sometimes separated from each other by commas.

5. Thus, beginning at the right, we have the *first period*, consisting of ones, tens and hundreds *of units;* the *second period*, ones, tens and hundreds *of thousands;* the *third period*, ones, tens and hundreds *of millions.* The *fourth period* is that of *billions;* the *fifth*, *trillions;* the *sixth*, *quadrillions;* each period containing ones, tens, and hundreds of that period.

11. Numeration Table.

Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of	Hundreds of	Tens of	Ones of
Quadrillions			**Trillions**			**Billions**			**Millions**			**Thousands**			**Ones**		
SIXTH			FIFTH			FOURTH			THIRD			SECOND			FIRST		
7	4	1	8	5	2	9	6	8	0	7	4	1	9	7	5	8	1
		7	2	9	9	9	7	7	8	0	2	8	1	4	3	5	6
			8	6	5	2	4	3	5	7	6	0	0	6	0	5	0
1	8	2	6	5	3	5	7	8	0	0	0	7	6	9	3	7	8
				3	1	8	4	0	9	9	9	0	0	0	6	4	2
	1	0	0	0	0	0	0	0	9	9	6	3	4	2	8	7	6
8	7	5	7	6	3	9	5	4	1	2	3	5	0	8	6	2	7

EXERCISES IN READING AND WRITING NUMBERS.

Read the foregoing numbers, consulting the headings, till you get accustomed to the names of the periods.

Read also as follows, or as may be directed by the teacher.

a. Read the tens' and ones' columns in the first period on the ight.

b. Read all the numbers of the ones' period.

c. Read the right-hand column in the thousands' period.

d. Read the tens' and ones' columns in the thousands' period.

e. Read the two right-hand periods.

f. Read the right-hand column of the millions' period with he left of thousands, as hundred thousands.

g. Read the millions' and ones' periods, omitting the thousands' period as if filled with ciphers.

Note.—These exercises may be varied to almost any extent.

1. *Read these numbers:*

10,000	83,000	60,000	75,000	150,000	756,000
23,600	14,900	46,300	65,100	294,000	632,480
47,225	83,720	85,493	62,340	392,500	290,405
80,027	90,008	84,003	60,050	576,168	161,002

2. *Write and read:*

800,000
743,000
875,670
135,791
450,000
200,000
237,090
987,654
790,000
359,200
684,210
500,007
206,960
941,000
678,800
246,890
410,000
562,387
143,576
635,794

3. Write and read 2,000,000; 5,000,000; 7,000,000; 4,000,000. Fill the places occupied by ciphers with any digits you choose, and then read the numbers thus formed. Do this in various ways.

4. Copy the following numbers, and read them; then erase the digit at the right hand, and arrange the periods anew, by placing the commas where they should be, and read:

1,635,987
416,429,863
134,764,211
7,763,664
29,876,354

Continue this exercise by erasing the digits one by one, and pointing off the periods correctly.

5. How many tens, hundreds, thousands, ten-thousands, hundred-thousands and millions are expressed in those places respectively, in the numbers of Exercise 4?

Review Exercises.

Hundreds.—**1.** Write 300 and 20 and 7 as one number, expressed by three figures. In the same way write:

400 and 50 and 3; 100 and 70 and 6; 200 and 80 and 2;
300 and 60 and 6; 600 and 90 and 4; 900 and 10 and 1.

2. Write:

1	hundred	4	tens	6	ones;	2	hundreds	6	tens	3	ones;
1	"	8	"	7	"	4	"	7	"	5	"
1	"	9	"	2	"	8	"	0	"	0	"

3. *Read:*

527
729
313
723
792
901
168
843
910
365
290
109
134
209
953
524
902
646
340
299
728

Thousands.—**4.** Make the following numbers larger by one thousand: 328, 456, 508. By three thousand. By five thousand.

5. What number is next greater than 1599, 3019, 4091, 8400, 6379, 4599, 9999, 8765, 9109, 3099, 4098?

6. What number comes next before 2000, 7000, 4600, 5060, 3010, 2790, 8970, 1000, 1010, 7801?

7. Count and write from 996 to 1006: from 3189 to 3200; from 7990 to 8012; from 3001 back to 2989; etc.

8. How many hundreds and tens in 43 tens? In 68, 37, 56, 27, 49, 168, 434 tens? How many in 386 units? In 468, 125, 632 units? In 354, 538, 624 tens?

9. How many thousands and hundreds are in 25 hundreds? In 61, 52, 47, 56 hundreds? How many in 250 tens? In 310, 161, 289, 364, 543 tens? How many in 6987 units?

Tens and Hundreds of Thousands.—**10.** Prefix first twenty, then sixty, then forty thousand to 438, 132, 596, 100.

11. What number next greater than 25,999? 130,109? 199,999? 888,889? 986,290? 18,400? 689,999?

12. What number next less than 300,001? 700,000? 147,000? 354,989? 500,790? 100,000? 600,999? 489,123? 500,000?

Definitions.

12. A ***unit*** is one of any order or kind.

13. A ***number*** is a unit or collection of units.

14. ***Notation*** is the expression of number by *figures* or *letters.*

15. ***Numeration*** is the reading of numbers written in letters or figures.

16. All the digits have a ***Simple Value*** and a ***Local Value.*** A simple value, when they represent units or ones; a local value, when used to express tens, hundreds, etc. This value is called local because it depends on *the place which the digit occupies* (its locality).

17. The nine digits are signs of number, hence they are called ***Significant Figures.*** In this sense, the cipher "0" **is not** a significant figure.

Roman Notation.

18. The following table gives a complete view of a method of representing numbers by letters. This is called the ***Roman*** method because first used by the Roman people.

Table.

Thousands.	Hundreds.	Tens.	Uults.
M	C	X	I
MM	CC	XX	II
MMM	CCC	XXX	III
$\overline{\text{IV}}$	CCCC or CD	XL	IV
$\overline{\text{V}}$	D	L	V
$\overline{\text{VI}}$	DC	LX	VI
$\overline{\text{VII}}$	DCC	LXX	VII
$\overline{\text{VIII}}$	DCCC	LXXX	VIII
$\overline{\text{IX}}$	CM	XC	IX

$\overline{\text{X}}$—10000

Note 1.—It will be noticed that in writing four and nine of each order, a letter of less value is placed before one of greater value. In this case the less value is deducted from the greater. Thus, XL (ten less than fifty) is written for XXXX. CD (one hundred less than five hundred) is written for CCCC, etc. This mode of abbreviation is common, not universal.

Note 2.—A bar over a letter, or combination of letters, increases its value a thousand times.

For writing numbers in Roman numerals, we have the following

19. ***Rule.*—Write the several terms in order as given in the table.**

EXERCISES IN THE ROMAN NOTATION.

1. Read XXIV, IX, XIX, XV, XIV, LX, XLIV, LXXXIX, XC, XCIX, CCI, CCCXCIX, CD, CDLVIII, CDLIX.

2. DV, CDXCIX, DXLVI, DCCCIX, CMXCIX, MD, MIX, LIX, DIX, MDCCCLXXXIV, $\overline{\text{MCDLXX}}$.

---

3. Write in Roman numerals, 54, 72, 83, 59, 119, 72, 38, 49, 63, 98, 75, 69, 43, 91, 108, 319, 444, 333, 991, 3847, 2563, 3482.

CHAPTER II.

ADDITION.

Examples.—**1.** A hunter shot 6 rabbits on Monday, 7 on Tuesday, 8 on Wednesday, but only one on Thursday. How many rabbits did he shoot?

2. Charles is 9 years old. How old will he be in 6 years? In 5 years? In 8 years?

3. Fred had 8 dollars in his bank; he received 7 more on his birthday, and 4 at Christmas. How much had he then?

4. Grandfather was 53 years old when his grandchild was born. How old is he now that his grandchild is 9 years old?

5. The sun rose at 6 o'clock this morning; that was 3 hours ago. What o'clock is it now? What o'clock 5 hours after sunrise? 4 hours? 6 hours?

6. William read 7 pages in the morning, 3 in the afternoon, and 4 in the evening. How many pages did he read that day?

7. Sarah goes up and down stairs 8 times in the morning, and 5 times in the afternoon. How many times in the day?

---

8. Count to one hundred.—Count by twos to 100.—Count by threes to 99. Count by fours to 100.—Count by fives to 100.—Count by sixes to 96.—Count by sevens to 98.—Count by eights to 96.—Count by nines to 99.

The pupil may first write the result of each successive addition, and afterward go through the exercise orally.

9. How many units in 10 twos? (Count by 2's till you find out.) How many in 10 threes? In 10 fours? In 10 fives? etc.

10. Draw lines upon your slate, so as to divide it like a checker-board, but make ten squares instead of eight in each row, and as you count by 1's, write the results in the first line of squares from left to right; as you count by 2's, write the results in the second line of squares; as you count by 3's, write the results in the third line, and so on.

Definitions.

20. *Addition* in arithmetic is a process of finding the sum of two or more numbers.

21. Signs.—1. The sign + is read *plus*, and indicates addition; thus, 5 + 3 means 5 and 3 more.

2. The sign = is read *equals*, or *is equal to;* thus, 5 + 3 = 8 is read, 5 plus 3 equals or is equal to 8.

ORAL EXERCISES.

Write on slate or paper these two lines of figures.

4, 7, 2, 8, 8, 5, 6, 3, 9, 2, 6, 6, 8, 7, 9, 5, 3, 3, 4, 2, 5,
5, 3, 7, 7, 5, 5, 4, 6, 9, 9, 8, 4, 4, 2, 3, 8, 5, 4, 9, 6, 7.

11. To each number represented add 2, add 4, add 6, add 8.

Caution.—Do not say 4 and 2 are 6, but speak only the results, as *6*, *9*, etc. In *13* (below) give results directly, as *11*, *9*, *10*, *16*, etc.

12. Add 3, add 5, add 7, add 9, to each one.

13. Add the first to the second; add the second to the third, etc., beginning at the left—beginning at the right.

14. Add each number in the lower line to the one above it, proceeding first from left to right, and then from right to left.

15. 4+5+6=
16. 5+6+7=
17. 6+2+8=
18. 4+9+3=
19. 7+4+9=
20. 2+3+4+5=
21. 3+1+4+8=
22. 4+7+6+3=
23. 9+3+2+3=
24. 7+4+6+3=
25. 4+4+4+4=
26. 6+3+6+3=
27. 5+5+5+5=
28. 7+2+7+2=
29. 8+4+5+4=

30. 6+5+3=
8+4+2=
9+7+4=
5+8+3=
7+4+7=

31. 5+4+8+2=
2+4+6+8=
3+5+7+4=
3+6+3+7=
8+1+9+2=

32. 5+3+4+2=
9+7+1+2=
6+4+4+3=
9+4+4+3=
4+9+3+4=

33.	**34.**	**35.**	**36.**
10+6=	20+9=	90+2=	20+2+4=
30+5=	50+7=	80+4=	40+3+2=
40+4=	70+5=	70+6=	60+4+5=
50+3=	90+3=	60+3=	80+5+2=
60+2=	30+2=	50+5=	30+1+7=
80+7=	60+8=	40+9=	70+2+6=

37–117. Add 1, 2, 3, etc., up to 9, separately to each number in each line. Observe the units of the results.

1	11	21	31	41	51	61	71	81
2	12	22	32	42	52	62	72	82
3	13	23	33	43	53	63	73	83
4	14	24	34	44	54	64	74	84
5	15	25	35	45	55	65	75	85
6	16	26	36	46	56	66	76	86
7	17	27	37	47	57	67	77	87
8	18	28	38	48	58	68	78	88
9	19	29	39	49	59	69	79	89

This may be done orally, or on the slate, thus:

(37.)		(117.)	
	1+1=		9+9=
	11+1=		19+9=
	21+1=		29+9=
	31+1=		39+9=
	41+1=		49+9=
	51+1=		59+9=
	etc.		etc.

118.	**119.**	**120.**	**121.**
32+9=	33+9=	87+6=	56+8=
43+8=	44+8=	78+5=	65+6=
54+7=	55+7=	45+8=	29+9=
65+6=	66+6=	54+7=	92+5=
76+5=	77+5=	65+9=	87+7=
87+4=	88+4=	56+4=	78+6=
91+8=	22+7=	37+8=	47+6=

122–127. Add 6 to 21, 18, 36, 48, 54, 63, 17, 82, 88. Add also 8; 4; 5; 7; 9.

128–131. Increase the numbers 14, 19, 23, 25, 48, 84, 56, 37, 64, 83, 52, 38, 90, 87, 75, 61, 47, 79, 39, 59, 27, 69, 89, by 4; by 6; by 8; by 10.

133–135. Increase each of the numbers 23, 35, 48, 64, 56, 37, 41, 90, 82, 52, 61, 73, 84, 59, 47, 36, by 3; by 5; by 7; by 9.

Direction.—The foregoing exercises should be so thoroughly practiced, both orally and in writing, that the pupil can announce the sum of any two numbers expressed by single digits as readily as he can read them.—If he has to count his fingers in addition, he can proceed but slowly. He might as well spell every word as he reads, or crawl on his hands and knees instead of walking. Again, if he says "9 and 7 are 16," he uses five words where one, "sixteen," would be better.

Applications.—**136.** An hour has 60 minutes, and a half-hour has 30. How many minutes are there in one hour and a half?

137. There were hanging on a Christmas-tree 10 oranges, 20 apples, 30 nuts, 20 sugar-plums. How many gifts in all?

138. There are at work in a factory 40 men on the ground-floor, 30 on the second floor, 20 on the third floor, and 7 in the office. How many men are at work in the factory?

139. Grandmamma is 60 years old, mamma 30, and I am 7 years old. What is the sum of our ages?

140. An overcoat costs 30 dollars, a coat 20 dollars, a vest 4, and a pair of trousers 8 dollars. How much does the whole suit cost?

141. A fisherman caught in his net 36 pike, 30 bass, and 10 trout. Can you tell how many fish he caught?

142. A butcher bought two calves; one weighed 53 pounds, the other 47. How much did they weigh together?

ORAL EXERCISES.

Direction.—In adding, do not say (see 1st example) 4 *and* 5 *are* 9 *and* 6 *are* 15, etc., but give results at once; thus, *4*, *9*, *15*, *23*, etc.

143–168. *Add by columns and lines.*

4+5+6+8+9+4+3+7=
3+2+1+9+7+6+2+8=
8+4+3+9+5+7+8+6=
6+6+8+3+7+4+4+5=
7+8+2+7+6+9+6+5=

9+6+8+2+7+5+9+3=
6+7+4+9+3+1+7+5=
8+5+3+6+2+8+4+7=
3+5+4+9+7+2+8+2=
3+4+2+5+3+1+6+2=

169.	170.	171.	172.	173.
10+40=	90+10=	20+70=	40+15=	56+30=
20+50=	80+10=	30+60=	30+26=	44+20=
30+40=	60+30=	40+50=	60+38=	38+60=
40+50=	70+20=	50+30=	50+47=	29+50=
50+30=	50+40=	60+15=	70+28=	14+80=
60+40=	40+40=	70+26=	40+48=	63+30=
70+20=	80+20=	80+17=	60+37=	74+20=
90+30=	20+60=	20+45=	70+46=	88+40=
50+50=	30+30=	40+41=	80+11=	30+53=

174. 30+20+10+30=
175. 20+10+30+20=
176. 10+30+20+30=
177. 40+20+30+10=

178. 30+15+20+15+10+3+ 7=
179. 10+25+30+10+20+2+ 2=
180. 20+25+10+15+10+3+14=
181. 15+25+30+10+12+4+ 4=

Suggestions.—If, from this point to the rule on page 28, the examples seem too difficult, they may be omitted, to be taken up under the rule, but let the oral work be carried as far as possible. The learner who is left to himself to work out all his exercises on the slate is apt to form habits fatal to accuracy.

Applications.—182. There are 25 girls and 23 boys in a schoolroom. How many pupils in all?

183. On one side of Blair Street there are 34 houses, on the other side, 59. How many on both sides?

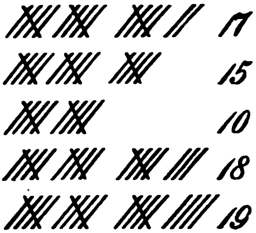
184. Mr. H. bought a horse for 73 dollars; he sold it and gained 19 dollars. For how much did he sell it?

185. Our house has 17 windows, that of one neighbor has 26, and that of another neighbor has 13. How many windows are there in the three houses?

Exercises.—186–206. Add rapidly, by columns and lines, giving results only:

4+6+4+8+7+4+9+4+3+5+2+3+1+8+9=
8+5+3+9+6+8+5+3+6+7+9+7+6+9+1=
4+9+5+6+9+7+6+2+1+8+3+4+2+3+9=
7+6+5+4+3+2+1+9+7+5+6+4+9+4+8=
4+3+2+1+2+3+4+5+6+7+8+9+7+6+7=
8+4+6+9+1+7+3+5+7+9+6+8+7+5+6=

Note.—Let the pupil illustrate examples 207 to 226 by the use of buttons, acorns, or other objects which he can tie into bundles or string together in collections of ten; or let him make marks upon the slate such as these at the right, designed to illustrate example 207.



Thus, understanding well the nature of the thing to be done, he will need no rule for the simple operations here required. Let him first add the tens, and to the sum let him add the numbers in units' place.

207. 17+15+10+18+19=
208. 14+16+18+20+13=
209. 12+13+20+14+16=
210. 20+19+11+18+13=
211. 18+15+14+17+16=
212. 12+14+16+13+15=
213. 15+17+19+16+14=
214. 19+12+13+17+16=
215. 17+18+14+17+18=
216. 20+19+19+12+13=
217. 12+15+ 9+10+13=
218. 18+10+17+ 9+18=
219. 19+17+15+13+11=
220. 13+20+ 7+16+12=
221. 23+17+18+ 2+15=
222. 8+12+13+19+21=
223. 11+17+28+ 6+18=
224. 27+19+ 8+20+16=
225. 12+24+18+ 6+17=
226. 20+13+16+10+25=

Suggestion.—Exercises in numeration should precede the following examples.

Applications.—**227.** There were at a party 50 gentlemen, 60 ladies, and 70 children. How many people were there?

228. A farmer raised 80 bushels of wheat, 39 bushels of oats, and 10 bushels of barley. How many bushels in all?

229. There are in an orchard 63 plum-trees, 75 apple-trees, and 11 peach-trees. How many trees in all?

230. A book-case has on the first shelf 48 books, on the second 57, and on the third 75. How many on the 3 shelves?

231. There are 68 boys in one room of a school-house, 73 girls in another, and 87 girls and boys in a third. How many pupils are there in the school?

232. The first book of Moses has 50 chapters, the second 40, the third 27, the fourth 36, and the fifth 34. How many in the 5 books?

Add without the use of the slate:

33. 50+60=
60+80=
70+40=
80+30=
90+50=
40+90=

234. 40+60+30=
50+30+50=
60+90+70=
70+40+90=
80+70+40=
90+50+60=

235. 80+38=
70+59=
90+67=
50+74=
60+83=
70+92=

236. 64+50=
76+60=
85+90=
59+70=
68+80=
75+40=

Note.—The examples in 235 and 236 require only one oral step, that is, the rect announcement of the result; as, for instance, in adding 80 and 38, think) and 30 (=110) and 8, but *say* at once ***118.*** In examples 237 to 241, two steps e enough; thus, in adding 59 and 32, first think 59 and 30, and say ***89,*** then 9 and 2, and say ***91.*** In examples 242 to 244, four steps may be necessary for e learner; thus, in adding 25, 38, and 49, say ***55, 63, 103, 112.***

237.
9+32=
4+27=
6+24=
7+25=
8+26=
5+37=
2+25=

238.
36+42=
47+33=
38+24=
58+25=
64+26=
47+27=
29+34=

239.
25+22=
24+35=
33+19=
42+35=
53+43=
65+26=
35+36=

240.
57+28=
28+39=
45+37=
36+38=
25+47=
34+39=
27+48=

241.
26+34=
24+16=
74+24=
25+33=
47+29=
58+33=
34+28=

242.
65+36+3=
85+74+5=
67+63+7=
58+54+6=
73+72+9=
68+45+4=

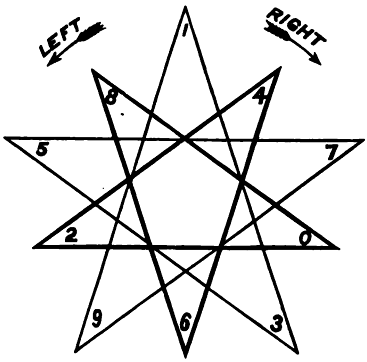
243.
85+47+5=
83+68+6=
94+42+9=
86+29+3=
92+56+5=
75+65+6=

244.
25+38+49=
97+23+65=
56+44+68=
83+28+25=
62+37+27=
56+48+39=

245–259. *Add by columns and lines:*

40+60+30+70+80+90+60+80+80+30
50+80+90+80+40+50+90+70+20+50
60+40+70+90+20+70+70+60+70+30
70+90+20+60+30+30+40+50+90+70
80+70+60+50+40+[illegible]+20+40+30+50

Suggestions for Blackboard Exercises.



22. In drill exercises, the double star affords some advantages over the circle, and at the same time facilitates the learning of the several series arising from successive additions of 2's, 3's, 4's, etc.

Direction. — Beginning at the unit figure of any given number, the unit figures of the successive sums will be found as follows:

1. In adding 3's, at the *next* point to the right, and so on; in adding 7's, at the *next* to the left.

2. In adding 6's, at the *second* point to the right, and thus on, from point to point, of the same star. In adding 4's, at the *second* point to the left, and so on.

3. In adding 9's, at the *third* point to the right.

4. In adding 2's, at the fourth point to the right, and thus on (following the line at the right of the last unit figure). In adding 8's, at the fourth point to the left (following the line at the left of the last unit figure).

5. In adding 5's, at the point directly opposite the unit figure of the given number, and thus to and fro.

23. Other Uses of the Figure.—A suitable number being written at the center, the numbers at the points can be combined with it, in addition, subtraction, multiplication, or division, as may be desired. The number at the center being changed from time to time, there is no end to the variety of exercises that may thus be had at little expense of time or labor on the part of the teacher. Exercises in common and decimal fractions may be given in the same way.

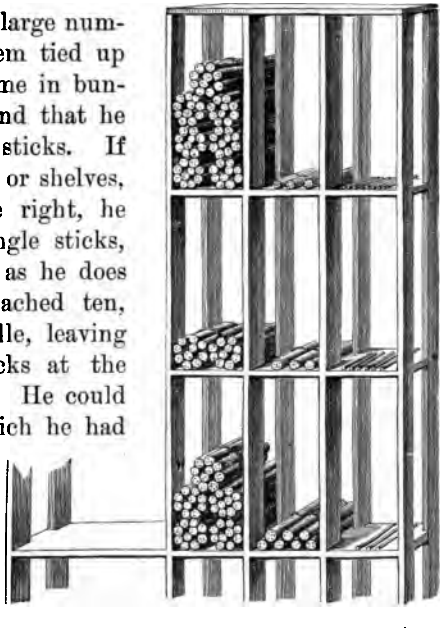
Addition of Higher Orders.

24. Note.—The illustrations of this book are not intended to be merely obrved and read about, but they are designed to picture to the eye, as far as possible, e actual work which it is intended shall be done by the pupils with objects. These jects should be supplied by the school authorities, or, with slight suggestions by e teacher as to what is best or most available, according to the circumstances of e school, they may be brought in by the pupils. They should be as large as posble, and yet not inconvenient to handle in great numbers.

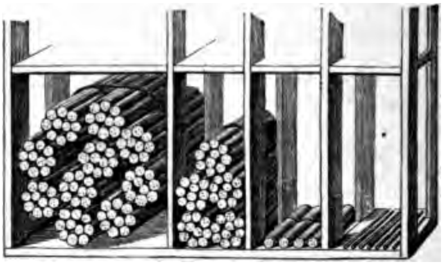
SLATE WORK.

Example.—Find the sum of 738, 236 and 573.

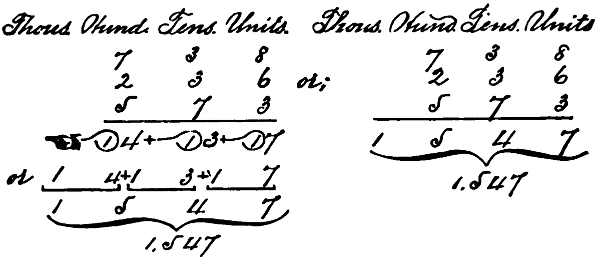
One who knows nothing more of arithmetic than how to count o ten might find the sum of these numbers by some such means s the following:



Suppose that he has a large numer of sticks, some of them tied up n bundles of ten, and some in bunles of a hundred each, and that he as, besides, some single sticks. If hese were placed in rows or shelves, s in the picture at the right, he night count first the single sticks, aking them in his hand as he does o, and when he has reached ten, ie them in a small bundle, leaving he remaining single sticks at the ight on the shelf below. He could hen count the bundle which he had ust made, with the tens' undles on the shelves, nd tie each ten of these undles into larger bunles of a hundred each, nd leaving the odd bunles of tens on the shelf below where they formerly were, he could count all the bundles of hundreds together. And, again, tying ten of these larger bundles into one, he would have the sticks arranged as here represented; that is, one bundle containing a thousand, five of a hundred each, four of ten each, and seven single sticks.



25. The foregoing method is the same as that which is indicated in the following arithmetical process:



Having seen how it is, that this process really produces a number equal to the sum of the numbers added, the pupil is prepared for the rule for addition.

26. ***Rule.*—1. Arrange the numbers to be added so that the figures of the same order shall stand in the same column, units under units, tens under tens, and so on.**

2. Begin at the lowest order, and add each column separately. If the sum of any column is less than 10, write it underneath. If it is equal to or greater than 10, place the right-hand figure of the sum under the column added, and unite the left-hand term or terms with the next column.

***Proof.*—In order to be quite sure that the addition is correct, add each column both upward and downward. If the two results are the same, there is little danger of error.**

SLATE EXERCISES.

Examples 1–11. *Find the sum of*

27	87	29	68	68	92	53	27	57	91	84
58	28	36	43	79	72	67	72	46	24	37
90	64	39	29	25	68	85	28	90	56	62
56	47	76	46	97	73	89	57	51	45	78

12–22. *Find the sum of*

48	83	69	21	43	96	81	49	36	85	20
47	59	56	53	28	45	47	57	17	24	38
56	62	67	38	57	51	63	42	58	66	47
29	48	58	78	46	23	86	56	32	77	93

23–77. *Add 345 to each.*

639	542	894	457	837	910	735	628	246	802	135
429	900	312	168	524	254	736	547	749	630	757
372	713	457	298	337	698	507	192	293	394	495
345	298	110	206	387	471	289	509	135	398	211
213	361	425	357	862	135	779	387	600	731	877

78–88. Add together the numbers in each column.

89–93. Arrange each line of numbers in column and add.

---

94. **Apples.**	**95.** **Nuts.**	**96.** **Oranges.**	**97.** **Peaches.**	**98.** **Lemons.**	**99.** **Plums.**	**100.** **Books.**
136	268	204	268	301	718	593
241	194	237	473	275	629	868
217	187	168	118	478	446	687
153	253	352	323	262	537	774
302	145	249	248	164	855	956

Add the following numbers, first arranging them in columns:

101. 125, 126, 138, 139, 140.

102. 87, 9, 55, 394, 225, 194.

103. 83, 194, 56, 168, 473.

104. 336, 195, 987, 9, 11.

105–114. *Add by columns. Also by lines.*

2123 + 2364 + 7025 + 428 + 20 + 2103 =
6854 + 2559 + 843 + 1125 + 359 + 23 =
698 + 1994 + 1427 + 2496 + 2478 + 437 =
1927 + 49 + 7917 + 6579 + 1000 + 3706 =

115–124. 3254+4015+7348+1570+ 439+7986=
968+ 916+3407+4630+1690+ 375=
725+1207+ 197+1820+ 420+ 9=
4302+ 885+8329+ 7+7756+8975=

The following may be solved first without the use of the slate. Only results should be pronounced. (In the last line, Ex. 127, for instance, say ***892, 952, 956.***)

125. 300+600=
400+900=
500+100=
600+400=
700+700=
800+200=

126. 600+70+38=
700+50+49=
800+80+76=
900+90+47=
300+60+83=
200+80+72=

127. 300+56+84=
400+72+65=
500+83+49=
600+64+57=
700+53+88=
800+92+64=

128–148. *Add by columns and by lines.*

568+487+2000+5872=
435+675+6060+9321=
357+894+7009+7234=
479+383+5800+5321=
692+596+3750+6947=
824+776+4680+3579=
546+484+3541+2468=

4769+634+2465=
1250+ 4+1975=
3456+ 27+ 888=
5861+ 5+ 935=
9642+ 95+ 576=
8347+ 7+6491=
4936+183+ 587=

Find the sum of

149.	**150.**	**151.**	**152.**
345,271	4,391,002	4,622,715	5,324,681
65,382	686,975	9,874,963	1,964,735
7,491	68	8,472,465	28,497
83,257	4,937	8,010,706	68
496,350	53,286	4,506,080	5,834
1,849	487,659	432,741	4,793
65,472	7,321,445	98,653	56,689

153. Add 768, 5,643, 12,354, 678,901, 5,847, 2,146,353, 975,321, 64,387,510.

154. Add nineteen, ninety, seventy thousand four hundred eight, 87, 1,625,847, 269,751, 3,894, twelve hundred sixty-one, 5,050,050, six hundred thousand six.

155. Add 198,725, 918,273, 1,928,370, 4,354,651, 34,234,534, 6,712,893, 647, 19, 1,345, 67,351.

156. Add 283,857, two thousand twenty, 998,722, five millions fifty thousand fifty, eight hundred thousand seven hundred twelve, 27, 192,875, 909,090, six hundred eight thousand four hundred ten, 34,827, fourteen hundred fourteen.

157.	**158.**	**159.**	**160.**
1,783	4,328	42,235	9,999,999
19,456	369,300	10,305,236	8,000,000
5,788	53,528	84,165,352	4,730,876
94,374	51,279	1,236,536	12,359,776
100,855	13,975	163,021	76,305,864
456,788	34,975	29,363,987	32,467,209
872,543	124,900	37,903,210	57,264,902
321,354	1,243,651	16,988,710	98,537,873

Direction.—In adding these columns, do not say (see Ex. 157) 4 and 3 are 7, and 8 are 15, and 5 are 20, and 4 are 24, etc., but simply speak results; thus: *4, 7, 15, 20, 24, 32, 38, 41.* The repetition of the numbers to be added increases liability to error.

Some can learn to add mentally numbers of even three places. Treating the tens and hundreds as units and tens (see note, p. 25), they would say, in Ex. 161, *27, 57, 65* tens = *650, 655, 661,* and set down the answer at once.

27. Adding two or more columns of figures at once is valuable practice in "mental arithmetic." It should be carried as far as time and the ability of the pupil will permit.

161–169.									
	275	369	876	629	483	987	797	519	357
	386	625	529	875	759	654	686	982	678

Examples for Practice and Review.

Applications.—**1.** I gave 83 marbles to Lewis, 34 to William, and 97 to Charles. How many did I give away?

2. In one book there are 89 pages, in another 246, and in a third 387. How many pages in all?

3. A certain tract of land was divided into four farms, one containing 113 acres, another 287, a third 180, and the fourth 320 acres. How many acres did the original tract contain?

4. There was a large number of cents in a bag. I took out of it first 289 cents, then 397, then 478, then 693, and then I found 134 cents left in the bag. How many cents did it contain at first?

5. Our school-house contains 6 rooms. In room No. 1 there are 25 boys and 31 girls; in No. 2 there are 18 boys and 29 girls; in No. 3, 21 boys and 37 girls; in No. 4, 29 boys and 19 girls; in No. 5, 45 boys; in No. 6, 53 girls. How many boys in our school? How many girls? How many children in all?

6. In a certain township there are six farmers. The first has 5 horses, 12 cows, 35 sheep, and 20 hogs. The 2d has 5 horses, 10 cows, 18 sheep, and 12 hogs. The 3d has 3 horses, 6 cows, 27 sheep, and 9 hogs. The 4th has 4 horses, 8 cows, 25 sheep, and 14 hogs. The 5th has 1 horse, 2 cows, and 6 hogs. The 6th has 8 horses, 17 cows, 45 sheep, and 27 hogs. (1) How many horses have the 6 farmers? (2) How many cows? (3) How many sheep? (4) How many hogs? (5) How many head of live stock has the first, the second, the third, the fourth, the fifth, the sixth? (6) How many head of live stock on the 6 farms?

For the solution of this and similar examples, follow the arrangement given here. It is called "Tabulating," or arranging in tables.

FARMERS	HORSES	COWS	SHEEP	HOGS	NO. HEAD
The First	5	12	35	20	
The Second	5	10	18	12	
The Third	3	6	27	9	
The Fourth	4	8	25	14	
The Fifth	1	2	—	6	
The Sixth	8	17	45	27	
Total					

Note.—If the sum of the footings in the last line is not equal to the sum of the extensions in the last column, the work is incorrect. Why?

7. Three farmers have fruit-trees as follows: The first, 72 apple-trees, 108 peach-trees, 18 quince-trees, 16 plum-trees, 19 cherry-trees. The second, 38 apple-trees, 219 peach-trees, 9 quince-trees, 27 pear-trees, 38 plum-trees, 3 cherry-trees. The third, 19 apple-trees, and 43 peach-trees. (1) How many trees of each kind? (2) How many fruit-trees has each farmer? (3) How many trees have all?

8. In the year 1880, Cincinnati had 255,139 inhabitants; Cleveland 160,146; Toledo 50,137; Columbus 51,647; Dayton 38,678; Sandusky 15,838; Springfield 20,730; Hamilton 12,122; Portsmouth 11,321. How many inhabitants in these 9 cities of Ohio?

9. Three farmers, last year, sold fruit as follows: The first, 79 bushels of apples, 391 bu. of peaches, 8 bu. of quinces, 39 bu. of pears, and 8 bu. of cherries. The second, 43 bu. of apples, 539 bu. of peaches, 19 bu. of quinces, 37 bu. of pears, 47 bu. of plums, and 6 bu. of cherries. The third, 27 bu. of apples, and 87 bu. of peaches. (1) How many bushels of each kind of fruit were sold? (2) How many bushels of fruit did each farmer sell? (3) How many bushels did all of them sell?

10. North America is inhabited by 54,566,936 people; the West Indies by 4,316,718; South America by 26,913,531; Europe by 311,694,029; Asia by 791,031,473; Africa by 199,921,600; Oceanica by 38,318,771. Find how many in all. (Statistics 1885.)

11. Three farmers divided their land as follows: The first had 6 acres in rye, 20 acres in wheat, 69 acres in corn, 2 acres in potatoes, 26 acres in meadow land, 19 acres were lying fallow, and 3 acres were in grapes. The second had 12, 39, 136, 5, 75, 26, 2 acres (take them in the same order). The third had 4, 19, 53, 19, 5, 4, 9. (1) Find, how many acres of each kind; (2) how many acres to each farmer; (3) How many acres in all.

12. Lake Superior has an area of 32,000 square miles; Lake Michigan 24,000, Lake Huron 20,400, Lake Erie 9,600, Lake Ontario 6,300. What is the total area of the five great lakes?

Original Problems.

28. **Note.**—Problems such as the first may be made up under the direction of the teacher with the aid of all the pupils of the class, due notice having been given of the kind of contributions desired. One problem per day, if possible, should be required of each pupil.

1. Find the number of pages read by all the pupils in books not used in school. (Each reports for himself, all set down the items and find the sum.)

2. Find how many examples all have solved within any given time; how many lines all have read; how many words all have spelled or missed; how many chestnuts, walnuts, hickory nuts, acorns, etc., all have gathered.

3. In a rural district an enumeration may be made of the number of horses, cows, sheep, hogs, chickens, etc.; of the apple, peach, cherry trees, etc.; of the eggs gathered, etc., etc., in the district, or on all the farms from which the children come. Let them report in writing, so that no sensitive child or parent be offended.

Note.—Such problems as the following should be written out by the pupils separately, and without the aid of the teacher. Paper, cut to uniform size, should be used for the purpose, and the exercises corrected, as in other language lessons.

4. Each pupil may imagine himself to be a store-keeper in any line of trade he pleases, and make problems in regard to it. He should write out each problem with the utmost care, and be sure that he knows the answer before he gives it to the class.

5. Questions may be prepared with the aid of a text-book in geography; as, for instance: What is the population of the —— largest cities of —— ? (the pupil's own state, the United States, any foreign country, or the world).

6. The teacher can at sight judge of the correctness of answers to such questions as the following: If there are ten wagons, and each one contains 23 bushels of apples and 27 bushels of potatoes, how many bushels of potatoes and how many bushels of apples in them all?

CHAPTER III.

SUBTRACTION.

Units and Tens.

Examples.—1. Fred had 13 cents, and bought a top costing 8¢. How many cents had he left? How many would he have had left if the top had cost 7¢? 6¢? 4¢? 5¢? 9¢? 3¢? (¢ is a sign for cents.)

2. In a class of 16 pupils there are 9 girls. How many boys are in the class?

3. Emily is 18 years old, and her brother is 9 years younger. How old is he? Their sister is 11 years younger than Emily. How old is the sister?

4. A little boy had 9 marbles, and bought enough more to make 15. How many did he buy?

5. What number must we add to 7 to make 14? 16? 19?

6. Beginning with 100 take away twos, till nothing is left. From 99 take threes, till nothing is left; from 100 take fours; from 100 take fives; from 96 take sixes; from 98 take sevens; from 96 take eights; from 99 take nines.

Definitions.

29. ***Subtraction*** is a process of finding what is left of a number when a part of it is taken away; also, of finding the difference between two numbers.

30. Terms used.—The number from which a subtraction is made is called the ***Minuend.*** The number which is subtracted is called the ***Subtrahend.*** What is left of a number after taking away a part of it is called the ***Remainder.*** The remainder is the ***Difference*** between the minuend and subtrahend.

Note.—*Minuend* means *to be diminished*, and *Subtrahend*, *to be subtracted.* Numbers added are sometimes called *Addends*, or, more properly, *Addenda.*

31. Signs.—The sign — (minus) placed between two numbers indicates that the number on the right is to be taken from the one on the left; as, 7—2=5, which is read, 7 minus 2 equals 5. ***Minus*** means ***less.***

ORAL EXERCISES. IN TENS AND UNITS.

8.	**9.**	**10.**	**11.**	**12.**	**13.**
6—2=	8—3=	7—4=	11—2=	12—5=	13—7=
26—4=	14—3=	35—2=	41—4=	42—3=	25—6=
54—2=	46—5=	27—6=	53—6=	76—7=	57—8=
78—6=	68—7=	59—8=	65—8=	64—5=	63—9=
88—8=	58—3=	63—2=	71—2=	52—3=	35—7=
92—2=	86—5=	75—4=	83—4=	26—9=	41—5=

14–103. From each of the following numbers subtract 3 as many times as you can. Subtract also 5, 7, 9, 2, 4, and 8.

10	20	30	40	50	60	70	80	90
11	21	31	41	51	61	71	81	91
12	22	32	42	52	62	72	82	92
13	23	33	43	53	63	73	83	93
14	24	34	44	54	64	74	84	94
15	25	35	45	55	65	75	85	95
16	26	36	46	56	66	76	86	96
17	27	37	47	57	67	77	87	97
18	28	38	48	58	68	78	88	98
19	29	39	49	59	69	79	89	99

---

104. Tell how many must be added to 4 (see line of numbers below) to make 7, how many to 7 to make 13 (the next higher number ending in the next figure), and so on.

4, 7, 3, 8, 7, 9, 6, 7, 4, 3, 0, 8, 6, 9, 5, 4, 5, 8, 6, 5.

Note.—Say 4 and *3* (=7), 7 and *6* (=13), 3 and *5* (=8), 8 and *9* (=17). Omit numbers in parenthesis.

105. From each number following subtract the sum of its digits; thus,

5+6=*11*, 56−11=*45*. Announce only results, *11*, *45*.

56, 44, 32, 53, 65, 87, 48, 29, 51, 73, 18, 92, 47, 64, 20, 70, 98, 85.

Applications.—106. Mr. Smith is 70 years old, his son is 40. How old was Mr. S. when his son was born?

How much greater is 7 tens than 4 tens.

107. Sarah had 30 cents, and her sister 50. How many more did the sister have than Sarah?

108. John has 20 marbles, William 60. How many more has William than John?

109. There are 52 houses on the east side of Linden St., and only 20 on the west, How many more on the east than on the west side?

110. A farmer sold 32 of his 92 sheep. How many had he left? How many would he have had left if he had sold 12 more?

111. One book has 84 pages, another 72. How many more pages in the first than in the second?

112. If you had 55¢ in your bank, how many would be left if you were to take out 15¢?

ORAL EXERCISES.

Speak only the remainders, thus, *97, 93, 88,* etc.

113. 100—3—4—5—7—9—3—5—5—7—2—7—5—8—3=

114. 99—5—2—3—8—6—5—7—8—2—3—5—6—9—5=

115. 95—2—8—3—2—3—1—4—1—3—5—7—6—5—4=

116. 87—3—5—7—9—8—6—4—2—3—1—4—7—6—8=

117. 98—7—3—4—7—2—1—2—5—2—8—6—9—4—6=

118–124. From 90, 80, 70, 60, 50, 40, 30 take 10, take 20, take 30.

125–131. From 67, 83, 42, 56, 95, 72, 61 take 20, take 30, take 40.

132–138. From 90, 80, 70, 60, 50, 40, 30 take 16, take 27, take 23.

139–145. From 67, 87, 47, 57, 97, 77, 67 take 37, take 17, take 27.

In Exercises 146–150 subtract first tens, then units. Speak only results.

146.	**147.**	**148.**	**149.**	**150.**
45—25=	61—21=	65—35=	64—54=	97—57=
54—84=	72—32=	76—46=	75—55=	55—35=
29—19=	88—53=	42—22=	86—46=	58—28=
67—37=	94—54=	37—17=	97—57=	63—33=
73—23=	45—15=	84—34=	58—48=	88—48=

In the following examples, subtract first the tens then the units, announcing only two results. Thus, in solving the first, *think* 30 less 10 and *say 20,* then think 20 less 2 and *say 18.*

151.	**152.**	**153.**	**154.**	**155.**
30—12=	20—18=	31—12=	37—19=	62—28=
40—14=	50—24=	42—16=	48—29=	73—36=
50—16=	40—35=	53—18=	54—28=	84—37=
60—18=	60—43=	64—15=	64—27=	95—39=
70—13=	80—24=	75—19=	76—39=	56—48=
80—15=	90—42=	86—17=	88—49=	68—29=
90—17=	70—21=	97—18=	96—48=	83—27=
100—19=	100—46=	98—19=	69—47=	94—48=

156–160. Subtract 11 from each of the numbers, 21, 31, 41, 51, 61, 71, 81, 91. Subtract also 13, 15, 17, 19.

161–164. Similarly from 24, 34, 44, 54, 64, 74, 84, 94 subtract 12, 14, 16, 18.

165–168. Similarly from 27, 37, 47, 57, 67, 77, 87, 97 subtract 19, 14, 18, 17.

169–224. Find the difference between each number and the one to the right of it in the same line.

99	82	73	69	58	42	36	24
91	87	75	63	52	38	25	12
93	89	69	55	47	32	21	14
95	81	79	67	53	41	39	21
94	88	71	66	57	44	27	13
99	77	68	56	41	35	24	17
92	88	77	66	55	42	37	19
97	85	73	61	50	45	35	28

225–230. From each one of the numbers 49, 52, 65, 76, 82, 93 take first 12, and from the remainder take 15. In the same manner take 13 and 16; 14 and 17.

231–236. Similarly from 41, 58, 63, 74, 85, 97 take 11, and from the remainder take 18; also 16 and 19; also 12 and 17.

Tens and Hundreds.

237.	**238.**	**239.**	**240.**
200—20=	500—50=	300—80=	700—60=
300—70=	600—40=	400=40=	800—30=
400—50=	700—30=	500—70=	900—50=
500—30=	800—20=	600—20=	300—60=
600—60=	900—10=	700—40=	400—80=

Subtract the hundreds, then the tens.

241.	**242.**	**243.**	**244.**
400—240=	800—420=	500—180=	600—280=
600—130=	500—450=	400—290=	800—370=
800—320=	700—260=	600—370=	900—580=
900—440=	900—310=	700—490=	600—390=
500—330=	300—220=	900—630=	700—210=

Applications.—245. Mr. A has 400 sheep, Mr. B 200. How many has Mr. A more than Mr. B?

246. A clerk receives a salary of $1,200, and pays $290 for rent. How much remains for other expenses?

Note.—The sign $ is used to denote dollars. It is called the ***dollar mark.***

247. Mr. Abel sold his house for $860. He had bought it for $730. How much did he gain?

248. If a boy resides 580 steps from the school-house, but, on going to school, stops on his way after taking 380 steps, how many has he yet to take?

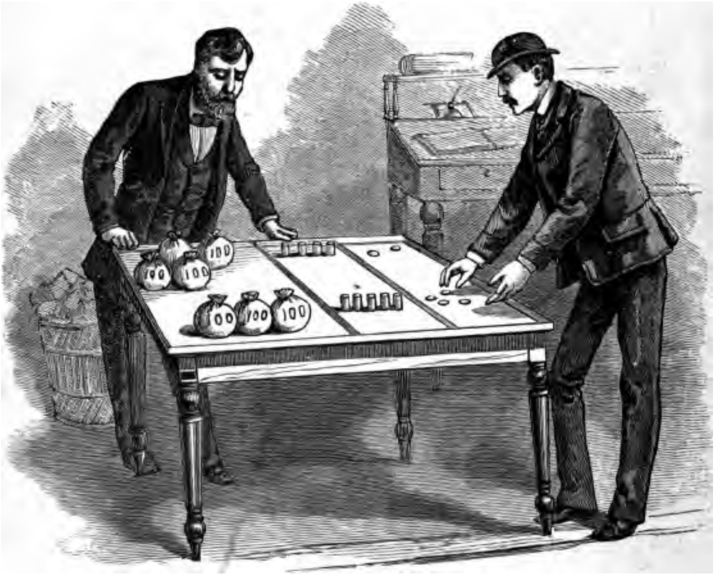
249. A farmer bought a horse and two cows for $195. If one cow cost $49, and the other $50, how much did he pay for the horse?

Units, Tens, and Higher Orders.

32. Case I. When no term of the subtrahend is greater than the term in the same order of the minuend.

Example.—From 796 subtract 354.

Illustration.—Suppose that Mr. Jones has seven sacks of money, each containing one hundred silver dollars, nine rolls of ten dollars each, and six dollars lying loose on his table, as represented in the following picture, and that Mr. Smith calls to collect 354 dollars. The pupil will readily understand that Mr. Jones has only to give Mr. Smith *four* of the single pieces, *five* of the ten dollar rolls, and *three* of the sacks containing one hundred dollars each, and that he will then have 442 dollars remaining.



Note.—The learner should practice himself in such illustrations till he is familiar with them. He will thus surely learn the significance of the processes of arithmetic.

Process of subtraction for slate work.

	Dollars.		Dollars.
Mr. Jones had	7 hundreds 9 tens 6 units.	Or,	796
Of this he paid	3 hundreds 5 tens 4 units.		354
He had left	4 hundreds 4 tens 2 units.		442

In the same way, tell how you would take 325 from 697 buttons, supposing that you had 6 cards having 100 buttons sewed on to each, 9 cards with 10 buttons on each, and 7 loose buttons. Show how the remainder would be found by work on the slate.

SLATE EXERCISES.

1.	2.	3.	4.	5.	6.	7.	8.	9.
495	589	686	798	894	995	789	795	325
182	273	325	496	472	572	364	682	124

10.	11.	12.	13.	14.	15.	16.	17.	18.
578	496	758	983	492	854	786	854	750
235	314	245	372	271	622	543	503	600

19–32. From 879 take 213, 425, 263, 34, 728, 658, 870, 457, 23, 43, 654, 222, 333, 400.

33–46. Take 432 from each of the following numbers: 543, 733, 645, 987, 655, 438, 679, 542, 632, 777, 989, 656, 686, 567.

Note.—In the process of subtraction there are two modes of reckoning. For instance, in subtracting 5 from 9, some say, "5 from 9 leaves *4*"; others, "5 and *4* are 9"; both writing the *4* as it is spoken, or better, as it comes into the mind. The results are the same, but the latter wording is recommended in practice for many reasons, one of which is that it is less liable to error. It should not be introduced, however, till the former is well understood by the learner.

Example.—From 789 subtract 435.

$$\begin{array}{r}789\\ \underline{435}\\ 354\end{array}$$

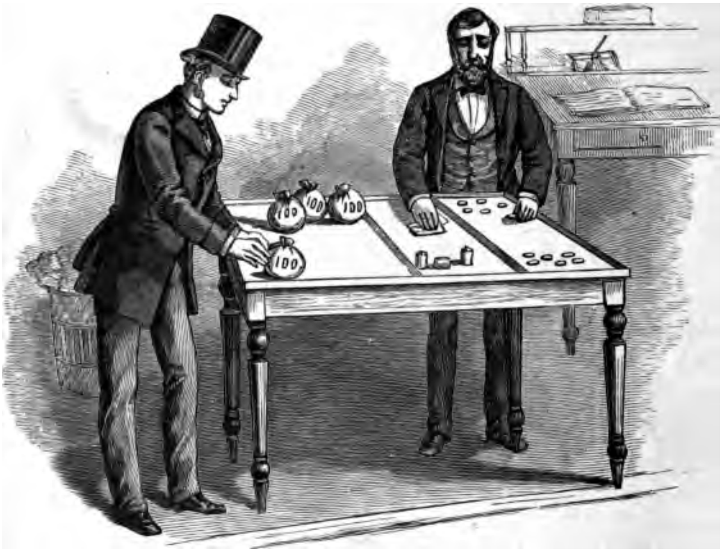
Wording.—5 and *4* are 9, 3 and *5* are 8, 4 and *3* are 7. Only the results printed in heavy type should be spoken, and these should be written as uttered.

This is called the "making up method." Besides being less liable to error, other advantages will be seen in Case II. See Ex. 68, p. 44, also Shorter Method in Long Division, p. 90.

33. Case II. When any term of the subtrahend is greater than the term in the same order of the minuend.

Example.—From 442 subtract 136.

Illustration.—After Mr. Jones had paid Mr. Smith, he had $442 left, out of which he is paying Mr. Brown $136. But since he has only two loose dollars, he is here represented as having taken ten dollars from one of the rolls, and put them with the two, thus making twelve single pieces. From these he has put forward *six* pieces. He has shoved forward also the *three* remaining ten dollar rolls, and *one* sack containing a hundred dollars. Thus he has —— dollars left.



87

The process is represented in figures as follows:

	Dollars.		Dollars.
Mr. Jones had	4 hundreds 4 tens 2 units.	Or,	442
Mr. Brown was paid	1 hundred 3 tens 6 units.		136
Mr. Jones had left	3 hundreds 0 tens 6 units.		306

2/24 1704 24 1706 168 26 24

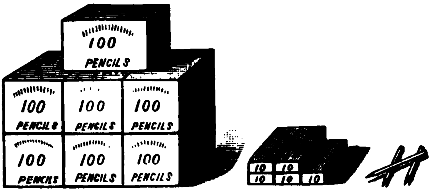
SLATE EXERCISES.

Subtract

47.	**48.**	**49.**	**50.**	**51.**	**52.**	**53.**	**54.**	**55.**	**56.**
324	543	384	792	325	986	847	761	568	738
116	327	259	368	217	509	719	234	139	309

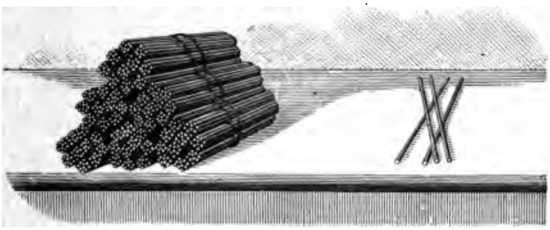
57.	**58.**	**59.**	**60.**	**61.**	**62.**	**63.**	**64.**	**65.**	**66.**
382	44	631	52	166	240	874	760	293	871
279	25	408	39	39	19	65	347	175	257

Note.—The pupil does not understand the foregoing illustrations if he can not go farther and explain the case, where he has to "borrow," both from the tens and from the hundreds; or, where there are no tens, perhaps also no units.



67. Explain how you could most conveniently take 585 pencils from the number of pencils represented above.

68. In the same way explain how you would proceed to take 378 match-sticks from the number represented below.



Arithmetical process:

There are	6 hundreds, 0 tens, 4 units.	Or,	604
We take	3 hundreds, 7 tens, 8 units.		378
There will be left	2 hundreds, 2 tens, 6 units.		226

According to the "making up method" mentioned in note at foot of p. 41, the wording in the process of subtracting 378 from 604 would be as follows :

$$\begin{array}{r}604\\378\\\hline 226\end{array}$$

Eight and ***6*** are 14, carry one to 7, 8 and ***2*** are 10, carry 1 to 3, 4 and ***2*** are 6, the numbers represented in heavy type being the only ones spoken aloud. These should be written while they are being pronounced.

69–78. Subtract 328 from each of the following numbers : 442, 560, 643, 751, 962, 876, 777, 691, 564, 886.

79–86. Find the difference between 435 and each of the following numbers : 561, 872, 960, 253, 864, 950, 762, 341.

Subtract

87.	**88.**	**89.**	**90.**	**91.**	**92.**	**93.**	**94.**	**95.**	**96.**
624	733	521	635	742	815	740	490	600	586
156	56	68	469	387	466	295	399	579	297

97.	**98.**	**99.**	**100.**	**101.**	**102.**	**103.**	**104.**	**105.**	**106.**
350	440	560	690	720	870	980	550	360	570
123	145	467	498	445	456	379	258	177	276

Try to solve these examples orally before using the slate.

107. 255−15=
108. 836−17=
109. 952−19=
110. 115−14=
111. 303−13=
112. 517−19=
113. 206−16=
114. 614−18=
115. 256−150=
116. 743−240=
117. 428−320=
118. 549−540=
119. 387−180=
120. 613−410=
121. 309−200=
122. 487−280=
123. 378−108=
124. 643−207=
125. 418−309=
126. 237−108=
127. 276−208=
128. 517−309=
129. 226−109=
130. 375−207=
131. 457−259=
132. 756−257=
133. 321−134=
134. 482−246=
135. 527−358=
136. 621−247=
137. 732−544=
138. 425−247=

Find the differences

139.	**140.**	**141.**	**142.**	**143.**	**144.**	**145.**
7834	9425	6453	7542	28769	52639	43892
4915	3568	2575	4634	3454	4758	5893

146.	**147.**	**148.**	**149.**	**150.**	**151.**
628234	23456	180020	70000	60000001	3845
149365	15897	147835	39876	12345678	1578

34. ***Rule.*—1. Write the subtrahend under the minuend, so that units may stand under units, tens under tens, hundreds under hundreds, etc.**

2. Commence at the right hand, and if possible subtract each term of the subtrahend from the one above it, and write the remainder below in the same order.

3. If any term of the minuend is less than the term to be subtracted, add ten to it, keeping in mind that the next higher term of the minuend must then be diminished by one.

***Proof.*—Add the remainder to the subtrahend. If the sum is equal to the minuend the work is correct.**

1907

Examples for Practice and Review.

1. How many years have passed since the discovery of America in 1492? Since the discovery of the Mississippi river in 1541?

2. Benjamin Franklin died in 1790, aged 84 years. In what year was he born? How old is a person now who was born in the year in which Franklin died?

3. A store, valued at $9,050, was destroyed by fire. What was the owner's loss, there being an insurance of $6,000?

4. There were 12,426 soldiers in a fortress, of whom 5,855 were discharged. How many remained?

5. A merchant began business with goods valued at $16,810. After two years he found his goods worth $38,430. How much had their value increased?

---

6. The State of New York had 5,082,871 inhabitants in 1880; the State of Illinois had 3,077,871. What was the difference in the number of inhabitants of these two States?

7. Asia is supposed to have 797,000,000 inhabitants, and Europe 313,834,000. How many more people are supposed to live in Asia than in Europe?

8. In 1880 there were 105,541 children attending the public schools of Philadelphia, 59,768 in Boston, and 270,176 in New York. How many more in Philadelphia than in Boston? How many more in New York than in Philadelphia?

9. Mt. Everest, in Asia, which is 29,002 ft. high, is 22,709 ft. higher than Mt. Washington, in New Hampshire. How high is Mt. Washington? ____________

EXERCISES FOR SLATE WORK.

10. From 750 subtract 75, from the remainder subtract 75, from the second remainder subtract 75, and continue this process of subtraction till you have subtracted 75 ten times.

11. From 4,590 take 459 ten times, as in the preceding example.

12. From 6,380 take 638 nine times. Before doing the work, say what the last remainder will be.

13. Write any number expressed by three figures, annex a cipher, and from the number thus formed take the first one ten times. Why is the answer the same as in 10–11?

14. Copy and solve example 237 (page 25); add the first, second, and third columns separately; subtract the sum of the second column from the sum of the third. Why should the remainder be equal to the sum of the first column?

Suggestion.—There will be no difficulty in stating *why*, if you lay out before you numbers of objects as represented in any one of the examples.

15. Copy and perform examples 118–121 (page 21), and subtract the sum of the first column from the sum of the third. Why should the difference be equal to the sum of the second?

Find the differences:

16.	7432	**17.**	8397	**18.**	9465	**19.**	7546	**20.**	4932	**21.**	5432	**22.**	6420
	2345		4567		7656		1667		2784		3765		2574

23.	4923	**24.**	6843	**25.**	7110	**26.**	8435	**27.**	9425	**28.**	4620	**29.**	7005
	4486		1876		3465		3583		2754		1629		1967

30.	9000	**31.**	50,000	**32.**	60,000	**33.**	90,000	**34.**	80,000	**35.**	1,000,000
	5793		14,312		24,635		45,678		39,876		493,624

36.	200,000	**37.**	700,000	**38.**	8,000,000	**39.**	1,000,000	**40.**	654,321
	146,231		102,009		4,563,921		912,345		200,000

41. 423,021 156,798	**42.** 524,632 243,738	**43.** 635,124 78,987	**44.** 543,210 244,567	**45.** 74,321,000 56,543,289
46. 5,246,312 1,472,536	**47.** 842,151 147,367	**48.** 624,001 175,548	**49.** 632,031 234,567	**50.** 73,500,493 12,345,678

51. 43,821 34,547	**52.** 54,312 34,343	**53.** 64,213 23,456	**54.** 75,314 62,345	**55.** 86,753 23,542	**56.** 840,170 654,398

57. 724,314 342,675	**58.** 842,531 123,654	**59.** 904,030 654,321	**60.** 800,012 187,654	**61.** 60,012,345 45,678,987
62. 8,421,303 3,544,534	**63.** 4,621,621 26,562	**64.** 725,321 46,845	**65.** 372,100 193,876	**66.** 743,628 100,000

67. 7,432,100 2,876,201	**68.** 85,731;465 74,635,679	**69.** 74,000,321 49,898,767	**70.** 9,333,122,210 7,457,935,767

Miscellaneous Examples.

Addition and Subtraction.

1. If the sum of two numbers is 36,251, and the greatet one is 26,659, what is the smaller number?

2. From the sum of 3,742 and 89,331, take the sum of 1,137, 2,065, and 3,820.

3. Add 74,321, 85,746, 25,100, 321,098; subtract 26,304, and from the remainder take 54,876.

4. Add the difference between 4,321 and 3,571 to the difference between 52,312 and 19,936.

5. From the difference between 533,016 and 154,693, subtract the difference between 19,876 and 17,987.

6. To what number must 893 be added four times to make 3,804?

7. From what number must 309 be subtracted five times to leave 173?

8. From what number must you take 8,763 to leave 3,849?

9. To what number must you add 89,650 to make 108,731?

10. How many times must 638 be taken from 7,280 to leave 900? to leave 262?

11. Fred. had 143 marbles. How many had he left after giving 19 to you, 25 to me, 38 to Paul, 49 to Edward, and losing 3?

12. A congregation had raised $78,596 for the erection of a building which was to cost $125,000. How much yet remained to be raised?

13. A florist had 3,746 tuberose bulbs. How many were left after selling 815, 150, 387, 479, and 1,091?

14. After a robbery a banker finds $1,657 in his safe. The evening before he had left in it $9,336. How much had been stolen?

15. In the year 1880 Chicago had 503,185 inhabitants, Cincinnati 225,139, St. Louis 350,518. 1. Find the sum. 2. How many had Chicago more than St. Louis? 3. More than Cincinnati? 4. How many had St. Louis more than Cincinnati?

The distance by rail from New York

To Albany is	142	miles.	To Chicago,	977	miles.
" Buffalo,	439	"	" Bloomington,	1,104	"
" Cleveland,	622	"	" Jacksonville, Ill.,	1,193	"
" Toledo,	735	"	" St. Louis,	1,285	"

16. By the aid of this table reckon the distance of Albany from each place named after it.

17. Also reckon the distance from Cleveland to Buffalo; to Chicago; to St. Louis. Also from St. Louis to Chicago; from Chicago to Buffalo.

Suggestion.—Write out a table, showing the distance from each city named in the foregoing list to the next. Make other problems from this or other tables of the kind.

18. A farmer had in his yard 31 chickens, 17 geese, 24 turkeys, and these, with his ducks, made up the entire number of his poultry, which was 97. How many ducks had he?

19. How many times can 93 yds. be cut from a piece of twine 385 yds. long? How much will be left?

20. A train started with 374 passengers. At the first station 16 left and 9 got on; at the second 11 left and 25 got on; at the third 3 left. How many passengers were on the train as it entered the fourth station?

21. In the six working days of a week a newsboy bought 76 papers a day, except Friday and Saturday, when he bought 10 more each day. He sold all but 3 on Monday, and 4 on Saturday. How many did he sell that week?

22. Bought a pair of ponies for $158, and sold them so as to gain $47. What did I sell them for?

23. Bought one pony for $100, and another for $76; paid $2 for shoeing each of them, and sold the pair for $210. How much did I gain?

24. A farmer has in one lot 53 beech, 87 maple, 18 hickory, 54 walnut, 28 poplar, and 327 oak trees. He sells all the walnut, 13 hickory, 78 maple, and 15 poplar trees for lumber. He cuts 281 oaks into railroad ties, and all the remaining oak and other trees for firewood. How many does he cut for firewood?

25. Bought a horse and carriage for $428, and in selling them shortly afterward lost $35 on the carriage, but gained $16 on the horse. How much did I sell them for?

26. On Monday Robert finds 43 eggs, 25 each on Tuesday and Wednesday, 26 on Thursday, 22 on Friday, 26 on Saturday. The next week he finds as many and 17 more. He sells 96 to a neighbor, and the cook uses 58. How many has he to send to market at the end of the two weeks?

27. The number of days in each month of the year is:

January,	31.	April,	30.	July,	31.	October,	31.
February,	28.	May,	31.	August,	31.	November,	30.
March,	31.	June,	30.	September,	30.	December,	31.

How many more days in the last 6 than in the first 6 months?

28. How many days in all the months which have *a* for the second letter of their names? In all the rest of the year?

29. How many days in all the months which have the letter *c* in their names? In all the rest of the year?

Original Problems.

35. Write problems for yourself and classmates.

Note.—The following skeleton problems may be used at first, if thought best.

1. —— goes to the store, buys — for —¢, and — for —¢. How much change does —— bring home out of —¢?

2. —— has —¢, buys —¢ worth of —, and — yards of tape, but forgets what she paid for it. She has —¢ left, and has lost nothing. What did she pay for the tape?

3. —— wished to buy —, which would cost —¢; had saved up —¢, and uncle would give —¢. How much more was still needed?

4. ——, ——, —— (three ladies), buy — bolts of muslin, each containing 39 yards. Mrs. —— takes — yards, Mrs. —— — yards, Mrs. —— — yards. They send the rest to a poor neighbor. How many yards does she receive?

5. —— has — inhabitants; —— has not so many by —. How many has the latter?

Hints.—Boys raise money for a foot ball; make contributions for buying an overcoat or pair of shoes for a poor schoolmate; cut ties for a railroad; buy and sell newspapers. The girls make squares for a quilt; cakes for a picnic. Father gets and pays out money. Compare the heights of mountains, distances of cities from each other, of places from the school-house, of the weight of a dozen boys with that of as many girls of about the same age, each giving his or her own weight. Ask parents for problems.

CHAPTER IV.

MULTIPLICATION.

1. How many marks are there here? (Count by 3's.)

/// /// /// /// /// /// /// /// ///

2. How much will 6 tops cost at 4¢ a piece?

Suggestion.—If you had learned nothing more of arithmetic than how to count by 4's, you could take one top and pay for it, and then another and pay for that, and so on till you had six tops, and had put down six piles of 4¢ each to pay for them. Then, counting by fours, you would find 24¢ to be the cost of the six tops.

3. In like manner with pebbles, acorns, grains of wheat, match-sticks, marks upon the slate, or other convenient objects, for counters, find the cost of 4 oranges at 7¢ a piece; also, separately, of 6, 8, and 9 oranges at 5¢ a piece.

4. In the same way illustrate how you might find the cost of 9 pounds of sugar at 5¢, at 7¢, at 10¢, at 8¢, at 6¢, at 9¢ a pound.

36. The process of counting in this way would become, in a short time, very tiresome. It would certainly be more convenient to learn, once for all, the sum of five 8's, than to have to find the sum every time we need to know it.

Finding the sum of 8 + 8 + 8 + 8 + 8, as we have already learned, is called ***Addition,*** but taking 40 for 5 times 8, without at the same time making the addition, is ***Multiplication.***

Having constructed a table which shows the sum of from one to ten 1's, 2's, 3's, 4's, etc. (see Ex. 10, p. 20), the pupil should now commit it thoroughly to memory.

For convenience it is here given in full, with the addition of the 11's and 12's.

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Directions for learning the Multiplication Table.—1. Multiply ***2*** (see figure at the head of the second column) by the numbers in the first column, repeating the two numbers and their product, thus: 2 times 2 are four, 3 times 2 are 6, 4 times 2 are 8, 5 times 2 are 10, etc. After learning a column in this manner, it will be well to reverse the order, and learn 2 times 2 are 4, 2 times 3 are 6, 2 times 4 are 8, etc.

2. The shading indicates the parts which are learned most easily. The real difficulties of the table are to be found in the unshaded parts. Special attention should therefore be given to them.

Definitions.

37. ***Multiplication*** is a short process of finding the sum of two or more equal numbers.

38. Terms used.—When we say 9 times 3=27, we ***multiply*** 3 by 9. The numbers that are multiplied together are called the ***Factors*** (makers) of the result.

Note.—Think of the word *factory*, a *place* where things are *made*.

39. The factor which is multiplied is called the ***Multiplicand.*** The factor which we multiply by is called the ***Multiplier.*** The result obtained, that is, what is produced by multiplication, is called the ***Product*** (the thing produced).

Note.—Twice a number is double that number, three times is triple, four times is quadruple, etc. Any number of times another number is a multiple of (that is, *many times*) the number. Hence, any product of a number is a multiple of that number. The multiplication table is a table of multiples.

Notice *multi* in *multi*ply, *multi*plicand, *multi*plier, *multi*ple. Multi means *many*, and ply, pli, or ple, means *fold*. Multiplier means *many folder*.

40. Signs.—The sign × is used to show that two numbers are to be multiplied together, as $4\times 7=28$ may be read, 4 multiplied by 7 equals 28, or 4 times $7=28$. The latter reading will be preferred in this book.

The product of two factors is the same, whichever is used as the multiplier; hence, in *performing* a multiplication, we generally use the one containing the fewest significant figures, because it is most convenient to do so; but, in *indicating* a multiplication, it is best for the learner to write that term before the sign (×) which properly comes before the word *times* in stating the reason for the multiplication.

SLATE EXERCISES.

5. After learning the table on page 52, complete the following table. Make other tables, changing the order of the factors.

	7	2	8	5	3	9	1	6	10
10	70	20	80	50	30	90	10	60	100
6	42	12							
7									
9									
3									
5									
8									

ORAL EXERCISES.

6. Write in order, *5*, *7*, *4*, *1*, *8*, *3*, *6*, *9*, *2*, *10*, and multiply each number by 2, by 3, by 4, etc., to 10.

Caution.—Do not say, 3 times 5 are 15, 3 times 7 are 21, etc., but, pointing at 5, 7, 4, etc., and knowing that you are to multiply each by 3, say ***15, 21, 12,*** etc.

7. Write the following lines of figures upon slate or paper:

4, 7, 2, 8, 8, 5, 6, 3, 9, 2, 6, 6, 8, 7, 9, 5, 3, 3, 4, 2, 5,
5, 3, 7, 7, 5, 5, 4, 6, 9, 9, 8, 4, 4, 2, 3, 8, 5, 4, 9, 6, 7.

Then, pointing successively between 4 and 7, 7 and 2, 2 and 8, etc., announce their products, thus: *28*, *14*, *16*, etc.

Note.—This affords an excellent exercise upon the multiplication table, within 10 times 10, and when the pupil can give the products almost as rapidly as he can speak, he is ready to go forward, and not till then.

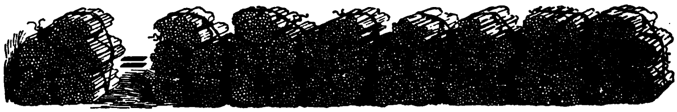
Seven times

7 × five tens are thirty-five tens.

Tens.—**8.** Multiply 90, 30, 60, 50, 80, 20, 70, 40, by 1; 2; 3; 4; 5; 6; 7; 8; 9.

Note.—Here the pupil should *think* 4 times 9 tens, but should say only, ***"36 tens,*** or ***360"***; ***"12 tens,*** or ***120,"*** etc.

Seven times



7 × five hundred are thirty-five hundred.

Hundreds.—**9.** Multiply 600, 300, 900, 200, 800, 500, 400, by 9; 3; 6; 8; 5; 2; 4; 7, and give the results, thus: ***54 hundred,*** or, ***5 thousand 4 hundred.***

Thousands.—**10.** Multiply 7000, 5000, 8000, 2000, 9000, 3000, 6000, by 8; 5; 3; 9; 4; 7; 2; 6.

SLATE WORK.

41. Units, Tens, Hundreds, etc.—**11.** Write 4685 nine times, as in the margin, and add.

We add thus: "5, 10, 15, etc., to 45." Then setting down the 5 units, and carrying the 4 tens to the column of tens, we say: "4, 12, 20, etc., to 76," and set down the six tens, carry the 7 hundreds to the next column, and add as before. Or,

Since all the figures in each column are the same, we save time by taking 45 at once for "9 times 5," as learned in the multiplication table. We then set down 5, and carry 4 to 9 times 8, and so on, exactly as if we were finding the sum by addition.

Addition.
4685
4685
4685
4685
4685
4685
4685
4685
4685
42165

Although 4685 is here written 9 times, as in addition, the last process by which we obtain the result is *Multiplication.*

In multiplying, however, we save more than the time required for counting by 5's, by 8's, etc. We save writing more than once the number to be multiplied, by simply noting the number of times each term is to be taken, as in the margin.

Multiplication.
4685
9
42165

Wording.—Knowing that you are to multiply 5 by 9, do not repeat "9 times 5 are 45," but say at once ***"45,"*** and write 5 in units' place as the word is pronounced. Then say ***72; 76,*** not "9 times 8 are 72 and 4 are 76." Use as few words as possible.

Examples.—**12.** What is the value of 9 acres of land at $783 per acre?

13. At $16856 per mile, what is the cost of constructing 8 miles of railway?

14. What will 6 horses cost at $273 each?

15. What is the cost of building 7 locomotives at $13586 apiece?

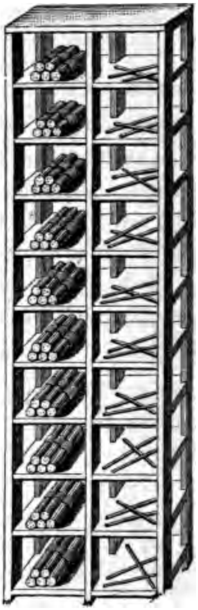
16. What is the value of 5 pianos at $785 each?

17–100. Multiply each of the following numbers by 6; by 9; by 3; by 7; by 4; and by 8:

4759	5678	2134	4157	5132	5426	3627
3846	8679	7986	9768	8979	6978	7896

Multiplying by 10, 100, 1000, etc.

101. Write 367 ten times and add. How many times 367 are there in the sum? How do the figures in the sum differ from the figures in 367?



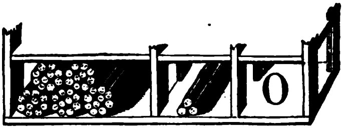
102. Write 3670 ten times and add. How many times 367 in 36700? How do the figures of this sum differ from those of 367?

103. Write 36700 ten times and add. How many times 367 in each 36700? How many times 367 in 367000? How do the figures of this last sum differ from those of 367?

Note.—The results in the last three examples indicate a principle which it is desirable the pupil should discover for himself. Additional examples of the kind may be dictated by the teacher.

42. The solution of the foregoing examples will enable the pupil to see that annexing one 0, thus removing the digits of a number one place to the left, increases the number tenfold; annexing two 0's to a number, thus removing its digits two places to the left, increases, or multiplies, its value *ten times* tenfold, or a hundredfold; that annexing three 0's multiplies the value of a number a thousandfold, etc.

Note on the Illustration.—One stick being taken from each of the right-hand boxes represented above, we have *ten*. These being tied together, are placed in the tens box below. Repeating this operation with the remaining single sticks, and proceeding similarly with the bundles of ten, it becomes evident to the senses that *ten times 53 are equal to 530.*



Hence we have the following

43. ***Rule.*—To multiply any number by 10, 100, 1000, etc., annex to the number to be multiplied as many 0's as there are 0's in the multiplier.**

ORAL EXERCISES

104. What will be the figures of the results if you increase each of the following numbers tenfold: 18, 92, 37, 802, 460?

105. Multiply 3562, 8921, 7643, 284, 39, 689, 9876, by 10.

106. Multiply 632, 54, 723, 140, 29, 3572, 60, 932, 807, by 100.

107. Multiply 15, 269, 387, 4, 5467, 198, 3287, 6420, by 1,000.

108. Multiply 6, 16, 26, 328, 10, 400, 632, 84730, by 10,000.

109. Multiply 71, 83, 94, 738, 8010, 4283, 738, by 100,000.

Multiplying by any Number of Tens, Hundreds, etc.

44. 110. Copy 𝍸 // 𝍸 // ten times, and show that 10 times 2 × 7 = 20 × 7. Copy 𝍸 // 𝍸 // 𝍸 // ten times, and show that 10 times 3 × 7 = 30 × 7.

111. If you were to copy 𝍸 / 𝍸 / 𝍸 / 𝍸 / one hundred times, you could show in like manner that 100 times 4 × 6 = 400 × 6.

112. How many times *five marks* are represented here? How

𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸

many *marks?* How many times *5 marks* in 10 such rows? How many *marks?*

In like manner tell how many times *5 marks* in 100 rows, and how many *marks.*

Suggestion.—Let similar exercises be continued till the pupil becomes so entirely familiar with the results that he can anticipate them with confidence.

---

113. Copy the following, and add by lines and columns.

16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16
16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16
16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16 + 16

How many 16's in each column? Three times 16 = ? How many columns are there? How many times 3 × 16? Ten times

3 × 16 = ? Ten times three 16's are how many 16's? Thirty times 16 then = ?

What difference between the figures in the products of 3 × 16 and 30 × 16? When you have found the product of 3 × 16, how can you most readily form the product of 30 × 16?

---

114. If you were to write three lines, each line containing one hundred 16's, and were to add the columns as before, the sum of each would of course be three times 16, and would = ? How many columns would there be? How many times 3 × 16? One hundred times 3 × 16 = ? One hundred times 3 × 16 are how many times 16? Three hundred times 16 = ?

What difference between the figures of the products of 3 × 16 and 300 times 16? When you know how many 3 × 16 are, how can you find 300 times 16 with least labor?

---

115. Copy the following line five times, setting the numbers carefully one under another, and add by lines and columns; then question yourself in the same manner as above.

327 + 327 + 327 + 327 + 327 + 327 + 327 + 327 + 327 + 327

Caution.—The learner should not neglect to perform all additions as directed.

116. Suppose you were to extend the five lines till you had a hundred 327's in each line?

Note.—You need not write them so often, but you may think of them as if they were written, and ask yourself,

How many 327's in a column? Five times 327 = ? How many columns would there be? A hundred times five times 327 = ? What is the difference between the figures of the product of 5 × 327 and of 500 × 327? Hence we have the following

45. ***Rule.***—**When the multiplier is expressed by a significant figure with ciphers annexed, multiply by the significant figure and annex as many ciphers to the product as there are ciphers at the right of the significant figure.**

ORAL EXERCISES.

117.	**118.**	**119.**	**120.**	**121.**	**122.**
20 × 8=	30 × 9=	60 × 3=	80 × 5=	40 × 6=	90 × 4=
20 × 5=	30 × 4=	60 × 7=	80 × 8=	40 × 8=	90 × 9=
20 × 4=	30 × 8=	60 × 5=	80 × 3=	40 × 5=	90 × 3=
20 × 7=	30 × 7=	60 × 6=	80 × 9=	40 × 7=	90 × 7=
20 × 3=	30 × 3=	60 × 4=	80 × 4=	40 × 3=	90 × 5=

123–130. Multiply 5, 9, 1, 8, 2, 7, 6, 4, 3, by 200 ; 500 ; 700 ; 900 ; 400. Also by 3000 ; 8000 ; 6000.

Note.—It may be better that the learner should not attempt the following exercises without the aid of the slate, but if he does, he will find it safest to multiply the tens first, increasing the right-hand term of that product by what he sees at a glance is to be brought up (carried) from the product of the units.

131. 20 times	**132.** 60 times	**133.** 40 times	**134.** 90 times	**135.** 50 times	**136.** 70 times
11	32	94	51	39	65
13	45	37	96	64	39
15	68	56	77	58	86
17	93	91	60	84	69
19	74	88	70	57	31

137–156.	8 × 12=	8 × 25=	6 × 33=	30 × 26=	20 × 48=
	5 × 19=	9 × 34=	8 × 19=	50 × 19=	70 × 13=
	8 × 24=	5 × 26=	7 × 14=	40 × 18=	50 × 17=
	9 × 18=	4 × 31=	9 × 12=	20 × 37=	40 × 24=

157–216. Multiply each number in the following columns by 3 ; 4 ; 5 ; 2 ; 6 ; 7 ; 8 ; 9.

217–276. Multiply them by 10 ; 20 ; 30 ; 40 ; 50 ; 60 ; 70 ; 80.

277–336. Also by 200 ; 800 ; 600 ; 500 ; and by 3000 ; 7000 ; 9000.

52	72	67	35	79	38	89	28	47	99
44	98	24	33	46	78	14	77	51	25
27	29	63	48	23	55	49	42	26	83
85	41	71	34	59	75	10	96	12	87
76	58	73	92	82	88	95	61	62	57
91	18	81	30	40	97	60	70	80	90

SLATE EXERCISES.

337–408. Multiply each number in the columns below by 9; 6; 2; 8; 4; 7; 5; 3.

409–480. Also by 30; 50; 70; 90.

481–552. Also by 300; 600; 800.—Also by 8000; 5000; 4000.

162	171	648	207	984	435	384	613	4212
288	304	621	318	945	119	855	533	2345
335	465	864	424	952	769	564	828	6789
316	568	954	154	238	854	403	261	1234
384	483	848	925	357	968	177	695	5678
266	612	302	193	476	166	324	785	9123
520	132	324	965	587	231	125	674	4567
423	660	216	164	693	219	662	753	8912

Note.—1. With a little practice, such calculations as are required above can be performed without the aid of the slate. Let the *wording* be the [illegible]mple announcement of results; thus,

2. In multiplying 162 by 9 the announcements should be ***9, 144, 1458.***—In multiplying 423 by 90 they would be ***36, 378, 3807, 38070.***

3. Though these are excellent exercises for practice, no learner should trust the result of oral work in large numbers without testing its accuracy by the written process.

Multiplying by Units, Tens, Hundreds, etc.

46. 1. Write 17 four times in one column, ten times in another, and fourteen times in a third. Add the columns separately.

Should the sum of the first two footings equal the third? Why?

2. Find 4 times 17 and 10 times 17 by multiplication, setting the work in the following form:

$$\begin{array}{r}4\times17=\ \ 68\\10\times17=170\end{array}$$

How many times 17 is the sum of the two products?

To find 14 times 17, then, let the slate-work stand as follows:

$$\begin{array}{r}4\times17=\ \ 68\\\underline{10}\times17=\underline{170}\\14\times17=238\end{array}$$

SLATE EXERCISES.

In like manner multiply:

3. 3 × 23 =
10 × 23 = ___

4. 5 × 31 =
40 × 31 = ___

5. 4 × 21 =
20 × 21 = ___

6. 8 × 63 =
30 × 63 = ___

7. 3 × 18 =
20 × 18 =
300 × 18 = ___

8. 6 × 71 =
50 × 71 =
600 × 71 = ___

9. 4 × 26 =
30 × 26 =
400 × 26 = ___

10. 2 × 46 =
70 × 46 =
200 × 46 = ___

11. 3 × 218 =
40 × 218 =
300 × 218 = ___

12. 4 × 324 =
20 × 324 =
200 × 324 = ___

13. 5 × 381 =
60 × 381 =
200 × 381 = ___

14. 7 × 1236 =
20 × 1236 =
100 × 1236 = ___

47. The use of many unnecessary figures can be avoided by writing the multipliers in their proper orders and as one number under the multiplicand, as in the second form below.

15. Multiply 3582 by 4376.

FIRST FORM.

$$\begin{array}{rcrr} 6 & \text{times} & 3582 = & 21492 \\ \mathit{70} & \text{“} & 3582 = & 25074\mathit{0} \\ \mathit{300} & \text{“} & 3582 = & 10746\mathit{00} \\ \mathit{4000} & \text{“} & 3582 = & 14328\mathit{000} \\ \cline{4-4} & & & 15674832 \end{array}$$

SECOND FORM

$$\begin{array}{r} 3582 \\ 4376 \\ \hline 21492 \\ 25074\mathit{0} \\ 10746\mathit{00} \\ 14328\mathit{000} \\ \hline 15674832 \end{array}$$

Though not nearly as many figures and signs are used in the second form as in the first, it must be remembered that both forms mean exactly the same thing. In both we multiply separately by units, tens, hundreds, etc., and then add the products. The products and the way of obtaining them are just the same in one form as in the other.

Definition.

48. When a multiplier contains two or more terms, the product of the multiplicand by any one of them is called a ***Partial product.*** (It is *only a part* of the entire product.)

SLATE EXERCISES.

Arrange the work in both forms.

16. 3 × 2371 =	**17.** 6 × 3582 =	**18.** 3 × 1356 =	**19.** 5 × 4026 =
40 × 2371 =	70 × 3582 =	40 × 1356 =	70 × 4026 =
500 × 2371 =	300 × 3582 =	500 × 1356 =	900 × 4026 =
2000 × 2371 =	1000 × 3582 =	2000 × 1356 =	3000 × 4026 =

49. The positions of the digits in each partial product sufficiently indicate their order; the ciphers at the right hand may therefore be omitted, if great care be taken to set the first figure produced by each multiplication under the figure by which you multiply, as in the example.

THIRD FORM.

```
    3582
    4376
   21492
  25074
 10746
14328
15674832
```

Note.—Frequently read the partial products, not forgetting the 0's.

20. 1776	**21.** 3028	**22.** 4958	**23.** 2888	**24.** 3675	**25.** 4788
48	52	74	36	49	57
26. 6045	**27.** 5186	**28.** 2415	**29.** 3456	**30.** 2886	**31.** 3306
65	76	35	26	47	38
32. 2106	**33.** 3569	**34.** 5076	**35.** 4940	**36.** 5412	**37.** 2401
79	41	58	65	438	492
38. 3536	**39.** 3436	**40.** 6882	**41.** 3884	**42.** 1102	**43.** 5605
521	635	749	362	293	643

44. Multiply 756 by 649; also by 351. How many times 756 in the sum of the products?

45–332. Multiply each number in the following columns by 589; by 376; by 4863; by 6974; by 892; by 3892; by 402; by 2009.

706	483	1858	3128	27228	42471
709	759	6696	5912	59432	60392
245	665	3054	6048	18737	47973
698	534	2968	3582	29735	46795
596	498	2505	2380	61356	12820
737	751	4731	3498	43207	55043

333–386. Multiply the numbers of the first column by 10; by 100; by 1000. Tell how you do it.—Multiply them by 30; by 400; by 6000. Tell how you do it.—Multiply them by 12; by 120; by 1200; and tell how it is done in each case.

---

50. The following is the general rule for multiplication:

***Rule*—1. If the multiplier consists of one term, multiply each term of the multiplicand by it, beginning at the right, and carrying as in addition. The result will be the product.**

2. If the multiplier contains more than one term, multiply separately by each term as above, placing the right-hand figure of each partial product in the same order or column as the term by which you are multiplying.

3. Add the partial products together. The sum will be the entire product.

***Proof.*—Multiply the multiplier by the multiplicand, and if the result is the same as before, the work is correct.**

***Contractions.*—If there are ciphers at the right hand of either factor, omit them in multiplying, and annex as many ciphers to the product as are thus omitted from both factors.**

Examples for Practice and Review.

1. How many letters in a book of 178 pages, if every page contains 2,978 letters?

2. The roof of the main part of a house has 128 rows of shingles, each row containing 129 shingles. How many shingles in that part of the roof?

3. How many lines are there in a work of 15 volumes, if each volume has 348 pages, and each page 46 lines?

4. There are 4 shelves in a drug-store, each containing 18 jars; 6 shelves, each containing 27 jars; and 5 shelves containing 12 jars each. Find how many jars in all.

5. Two school-houses were to be furnished with single desks. One had 18 rooms, each large enough for 52 desks; the other had 14 rooms, each large enough for 64 desks. Find how many desks were required for both houses.

6. A contractor ordered 52 loads of brick of 1,250 each. How many bricks did he order?

7. Multiply any number by 3, by 5 and by 2, and add the products. Why is the sum the same as the product would be if the number were multiplied by 10?

Multiply as indicated below, and find the sum of each column of products. Tell why the sum in each case contains the same digits as the several multiplicands of the example.

8.	9.	10.	11.
4 × 327 =	5 × 384 =	3 × 9876 =	7 × 48 =
2 × 327 =	3 × 384 =	3 × 9876 =	2 × 48 =
4 × 327 =	2 × 384 =	4 × 9876 =	1 × 48 =

Perform the following multiplications, and add the column of products in each example.

12.	13.	14.	15.
18 × 356 =	15 × 438 =	27 × 967 =	23 × 9807
19 × 356 =	17 × 438 =	32 × 967 =	14 × 9807
26 × 356 =	49 × 438 =	12 × 967 =	15 × 9807
37 × 356 =	19 × 438 =	29 × 967 =	48 × 9807

Compare the sum of products in each case with the several multiplicands of the same example. What do you observe? Why do you think such a result is found?

Suggestion.—Add the column of multipliers in each example.

Perform the following multiplications, add the products as in the preceding examples, and tell what you can about the results.

16.	17.	18.
103 × 587 =	292 × 8264 =	387 × 5379 =
369 × 587 =	375 × 8264 =	203 × 5379 =
287 × 587 =	109 × 8264 =	141 × 5379 =
241 × 587 =	224 × 8264 =	269 × 5379 =

•**Question.**—Is there not a shorter way to find the sums of products as required *on this page?*

Exercises in Familiar Measures.

51. There are 12 inches (in.) in 1 foot (ft.), 3 feet in 1 yard (yd.).

Note.—If the pupils are not already familiar with these measures, they should become so as soon as possible. They should be required to measure the length and the height of desks, the width of windows, doors; the length and width of the room, of the window-panes, etc., etc. Let the learner become familiar with the weights and measures here presented, but it is not desirable that he should learn any tables at this stage, except by observation and experience.

19. Find how many feet there are in 46 yards.

Analysis.—Since there are 3 ft. in 1 yard, there are 46 times 3 ft. in 46 yards.

$$\begin{array}{r}46\\3\\\hline 138\end{array}$$

20. How many inches in 16, 79, 63, 479, 200, 571, 325 feet?

21. How many inches in one yard? In 2, 7, 9, 19, 34 yd.?

22. Find first how many feet, then how many inches, there are in 12 yards.—In 24, 36, 112, 324, 420, 500 yd.

---

52. There are 16 ounces (oz.) in a pound (lb.), and 100 pounds in a hundredweight (cwt.). (The *c* stands for *hundred*, and *wt* for *weight*.)

23. How many ounces in 3, 9, 16, 32, 87 pounds?

24. How many ounces in one hundredweight?—In 2, 3 cwt.?

25. How many pounds in 6, 8, 3, 12 cwt.?

26. How many pounds do you weigh? How many ounces? How much would you be worth at $125 a pound?

---

53. There are 2 pints (pt.) in a quart (qt.), and 4 quarts in a gallon (gal.). (Used for measuring liquids.)

27. How many pints are there in 2 quarts?—In 6, 8, 10, 21, 130, 425 qt.?

28. How many quarts in 6 gallons?—In 14, 35, 47, 81, 230, 653 gal.?

29. How many pints in one gallon?—In 2, 6, 13, 27, 35, 87, 237 gal.?

30. How many pints in a barrel containing 30 gallons?

54. There are 8 quarts (qt.) in a peck (pk.), and 4 pecks in a bushel (bu.). (Used for measuring fruit, grain, etc.)

31. How many quarts in 2 pecks ?—In 8, 15, 22, 49, 83, 114 pk.?

32. How many quarts in 1 bushel ?—In 7, 19, 68, 27, 346 bu.?

33. How many pecks in 9, 11, 59, 90, 170, 232, 566 bu. ?

34. Tell first how many pecks, then how many quarts, there are in 16, 93, 78, 136, 458 bu.

---

55. There are 60 minutes (min.) in an hour (h.), 24 hours in a day (d.), and 7 days in a week (wk.).

35. How many minutes in 3 hours ?—In 14, 21, 17, 23 h. ?

36. How many minutes in 4 h. 56 m.?—In 16 h. 30 m. ?

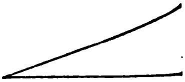
37. How many hours in 2 days ?—In 16, 15, 6, 9, 43 d. ?

38. How many hours in a week ?—In 2 wk. 3 d.?—In 3 wk. 2 d.?

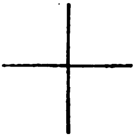
Definitions.

Note.—In the solution of some succeeding problems, the learner will need to know the following definitions:

56. The difference in direction of two lines is an ***angle.***

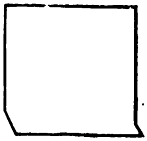


57. If the difference of direction is such that two lines, crossing each other, make four equal angles, the angles are ***right angles.*** (Right angles make square corners.)



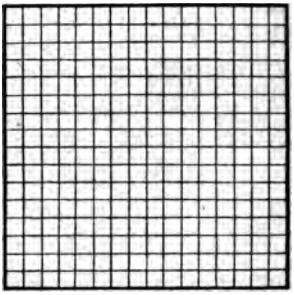
58. A ***rectangle*** is a figure having four straight sides and four right angles.

59. A ***square*** is a figure that has four straight and equal sides and four right angles.



A square inch is a square an inch long and an inch ***wide. A square foot*** is a foot long and a foot wide.

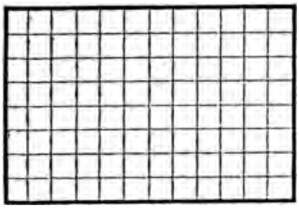
39. How can you find the number of small squares in the larger one without counting them singly? How many are there? How many would there be if it were twice as long and twice as wide?



Note.—For a home exercise let the pupils cut squares of paper measuring a yard, a foot, an inch, on each side, and of other required sizes. There should be a yard-stick in every school-room, and every student of arithmetic should have a foot-rule.

40. How many square pieces, each an inch long and an inch wide, can be cut from a piece of paper that is 4 inches long and 4 wide?

41. How many pieces, each a yard long and a yard wide, in a piece of oil cloth 6 yards long and 6 yards wide?



42. How many small squares in this rectangle? How many square inches would it contain if it were 7 inches long and 8 inches wide?

43. How many square feet if it were 15 feet long and 14 feet wide? How many square yards if it were 7 yards long and 3 wide?

44. How many square inches could you cut from a piece of silk 19 inches long and 12 inches wide?

45. How many square feet are contained in a rectangle 13 feet long and 9 feet wide? 58 feet long and 28 feet wide?

46. How many square inches in a square foot? (A square foot is 12 inches long and 12 inches wide.) How many in a square yard? (A square yard is 36 inches long and 36 inches wide.)

47. How many square feet can be cut out of a newspaper that is 4 feet long and 3 feet wide? How many square inches could be cut out of it? (See preceding Example.)

Miscellaneous Examples.

1. What is the product of 105, 106, 107 and 108?

2. What must be subtracted from 72×763 to leave 34×127?

3. What must be added to 47×436 to make 67×832?

4. Add 24×8,277 and 14×1,436, and from the sum subtract 8,763. What is the remainder?

Perform as many of the following examples as may be directed, and notice the curious results. Notice the sum of the digits in the several multipliers.

5. 45 × 987,654,321=
6. 45 × 123,456,789=
7. 54 × 123,456,789=
8. 54 × 987,654,321=
9. 27 × 987,654,321=
10. 72 × 987,654,321=
11. 36 × 123,456,789=
12. 36 × 987,654,321=
13. 63 × 987,654,321=
14. 63 × 123,456,789=

Note.—The intention here is not to teach the properties of the numbers, but to afford practice in multiplication. The products may awaken curiosity.

15. From the sum of 8,723, 57, 218, 9,658, 16, and 139, take the difference between 9,165 and 14,320, and multiply the remainder by 167.

16. Mr. Arnold and Mr. Bayard set out from the same place to travel in the same direction. Mr. A. travels 17 m. a day, and Mr. B. 20 m. How far apart will they be in a week (6 days)?

17. If the minuend is 16, and the remainder 7, what is the subtrahend? If the minuend is 90,087, and the remainder 26,089, what is the subtrahend?

18. Three boys together buy a peck of apples for 30¢; A. pays 9¢, B. pays 8¢, what does C. pay? Four men are in partnership with $16,876, of which A's share is $3,421, B's $2,500, and C's $5,693. How much is D's share?

19. An officer has an annual salary of $1,000; he spends $75 a month. How much does he save in one year? In 7, 9, 15, 17 *years?*

20. A skilled workman earned $3 a day; he worked 304 days in one year. How much did he earn that year? He spent $15 a week for himself and family. How much did he lay by?

21. A pipe pours into a reservoir daily 13,410 gallons of water. How many gallons in 30 days?

22. There are a dozen dozen in a gross, how many pencils in 75 gross?

23. A drover buys 12 horses at $85 apiece, 4 oxen at $68 apiece, 35 cows at $56 apiece, 237 sheep at $4 apiece. How much does he pay for all?

24. Mr. Ambros sells Mr. Dix 516 boxes of soap at $4 a box, and buys of Mr. Dix 389 bags of flour at $5 a bag. How much does Mr. Dix owe Mr. Ambros on the transaction?

25. If two boys can dig a row of potatoes in an hour, how long ought it to take one boy to do the same work? A piece of work can be done by 2 men in 7 days. How long would it take one man to do it?

26. A piece of work which can be done in 7 days by 45 men has to be performed by one man. How long will it take him?

27. There are 24 rows of trees in an orchard, and 24 trees in a row, how many trees are there? If there were twice as many rows and twice as many trees in a row, how many trees would there be?

28. I bought an overcoat, a vest, and a hat for $48. The overcoat and vest cost $42, the overcoat and hat $41. How much did each of the three articles cost?

29. Eleven hogsheads of sugar weigh respectively 918, 923, 891, 1022, 976, 889, 1019, 948, 901, 990 and 1080 pounds. How much was the weight of the lot less than it would have been if they had weighed 1086 pounds each?

30. How many toes have 238 camels, 86 ostriches, and 453 canaries. **(Let the pupil take time to find out what he needs to know to perform this example.)**

Original Problems.

To be Composed by the Pupils for the Class.

60. 1. Give the prices of things which you have bought, or which have been bought of you, supposing yourself to be a salesman in a store, at a fair, etc., and ask the cost. **(What must you give besides the prices?)**

2. Problems almost without number can be made about the number of square feet or yards in the floors, ceilings, blackboard, wall-maps, etc., of the school-room, giving the pupils time to measure for themselves.

Note.—The pupils proposing such questions should first take the necessary measurements. Let them take the nearest *whole* number of feet, inches or yards, according to the length of the line measured.

3. Give the number of rows of seats and the number of grown people that can sit in one row, and require the class to find how many can be seated in any church or hall you may name.

4. Borrow a tape-line, measure for yourself, and give the distance from one telegraph-pole to another, and tell the number of poles between any two places you may name; then ask how many feet or yards, from one place to the other.

5. Ask how many trees in Mr. ——'s orchard, after telling the class how many rows there are, and how many trees in a row. How many hills of corn in a field, etc.

6. Require to know *about* how many apples there are in a wagon-load of 50 bushels, say of greenings or any common fruit sold at market. If the class can't tell how it may be done without counting all the apples, tell them.

7. Give such problems as these, to be done in the shortest possible time, changing the numbers from those given here: What is the difference between 8 and 9 times 562? 35 and 45 times 976? Between 132 and 232 times 78? The sum of 36 times and 64 times 84? **(Be sure that you see the point yourself.)**

CHAPTER V.

DIVISION.

1. Write the letter *a* twenty-four times on slate or paper. How many times 12 a's are there in the 24 a's? How many times 6 a's? How many times 4 a's?

2. At 9¢ apiece, how many lead-pencils can be bought for 72¢? At 12¢ apiece? At 8¢? At 6¢?

Suggestion.—If a boy had 72 one-cent pieces, and knew no more of arithmetic than how to count, he could divide the 72 pieces into lots of 9¢ each, and thus find that for 72¢ he could buy 8 tops at 9¢ apiece, or as many tops as there are times 9¢ in 72¢.

3. With 42 ears of corn, how many horses can be fed if 6 ears are given to each? (Make 42 marks, and divide into groups of 6.)

4. In the same way, show how many dozen there are in 84, in 60, in 96, in 48. (Twelve single things make a dozen.)

5. How many top-strings can be cut from 48 feet of twine, if the strings are made 6 ft. long? If 8 ft. long?

61. A thorough knowledge of the multiplication table supersedes the necessity of marks, or other counters, except for purposes of illustration; for, if we know that twice 12 are 24, we know equally well that in 24 there are two 12's.

Note 1.—For practice at this point let a table, like the one suggested on page 53, be written on the black-board, the first column being omitted. Then a pupil pointing successively to each number in a column, and knowing that the number at the head is one factor, he announces the other; thus under 7 he announces *10, 6, 7,* etc., as rapidly as possible.

Note 2.—Here and elsewhere let counting be absolutely prohibited, except in the way of *illustration.*

ORAL EXERCISES.

62. Tens and Units.—*Caution.* In the following exercises do not repeat 3 in 18 six times, 3 in 30 ten times, but knowing that you are to tell how many times 3 there are in 18, 30, 27, etc., say at once ***6, 10, 9,*** etc.

How many times

6. 3 in	**7.** 4 in	**8.** 5 in	**9.** 6 in	**10.** 7 in	**11.** 8 in	**12.** 9 in
18	28	45	54	63	72	81
30	16	30	36	35	40	45
27	32	15	18	49	56	54
21	24	35	42	21	64	63
24	36	40	24	56	32	36
12	48	25	48	28	48	27
15	40	50	60	42	80	72

ILLUSTRATIVE EXERCISES.

Note.—The illustrative exercises introduced here and elsewhere are intended only as examples of what should be done in this direction. Problem after problem should be illustrated till the learner *feels* that in dealing with figures he is dealing with representatives of number, and that the arithmetical processes only indicate what a person ignorant of arithmetic might do to solve similar problems. Let the work be actually performed whenever possible. Labor impresses its lessons more deeply than observation. Mere instruction is not to be compared with it.

63. Hundreds.—How many times 9 in 270?

Write the letter *c* 27 times in a line, and mark them off into groups of 9 each; thus,

c c c c c c c c c, c c c c c c c c c, c c c c c c c c c.

Now, think how many c's there would be in *ten* such lines. How many times 9 c's. If you can not think the answers to these questions without writing the ten lines, write them and count.

64. Thousands.—How many times 4 in 3600?

Write the letter *e* 36 times in a line, and mark them off into groups of 4 e's each; then, think how many e's there would be in 10 such lines; in 100. How many groups of 4 in one line? *In 10 lines?* In 100 lines?

ORAL EXERCISES.

How many times

13. 3 in	**14.** 6 in	**15.** 7 in	**16.** 8 in	**17.** 9 in
210	480	560	480	450
240	360	490	400	630
270	420	630	720	810
150	300	420	880	540
180	540	700	640	720

How many times

18. 6 in	**19.** 7 in	**20.** 8 in	**21.** 9 in
4800	1400	2400	1800
5400	5600	8000	3600
3000	6300	4800	6300
1200	2800	6400	4500

How many times

22. 4 in	**23.** 6 in	**24.** 7 in	**25.** 8 in	**26.** 9 in
32	240	2100	6400	450
160	42	35	5600	63
240	5400	490	64	8100
3600	480	42	720	5400
80	4200	6300	80	720

27. How many times 10 in 80? 110? 70? 120? 60? 90? 20?

28. How many times 11 in 22? 121? 55? 132? 88? 66? 33?

29. How many times 12 in 84? 144? 72? 108? 60? 96? 132?

Applications.—**30.** How many dozen in 720? 600? 8400? 13200?

31. An army of 96000 men is marching 12 in a rank. How many ranks are there?

32. If pork is worth $7 a cwt., how many cwt. can be bought or $6300 ? $7700 ? $5600 ? 840 ?

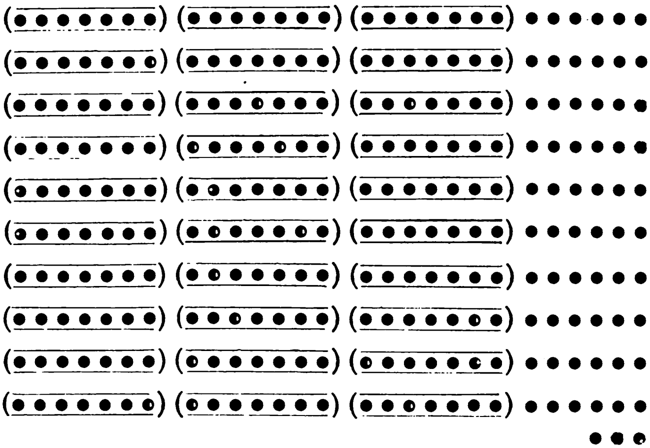
33. If a sheet of paper is folded so as to make 8 pages, how many sheets will be needed for 1600 pages ? 5600 ?

34. If a man walks at the rate of 4 miles an hour, in how many hours will he walk 48 miles ? 3600 miles ? 400 miles ?

ILLUSTRATIVE SLATE EXERCISES.

65. Thousands, and lower orders.—35. How many balls of twine can be bought for $2.73, or 273¢, at 7¢ a ball? (How many times 7¢ are there in 273¢.)

Make ten lines of 27 dots each, and beneath them make 3 single dots. Divide each full line of dots into groups of seven, in this way:



When the above work is neatly done, copy the following paragraphs, carefully filling all the blanks:

1. In each full line there are —— dots, which make —— groups (of 7), and — dots over.

2. In the ten lines there are — times as many dots, or —— dots; and — times as many groups, or —— groups; and there are also — times as many over.

3. The ten 6's over and the 3 in the short line beneath make together — dots. Out of 63 dots we can make —— groups of 7 dots each. Hence there are —— times 7 dots in 273, and —— times 7 cents in 273¢. Therefore, at — a ball, we can get — balls of twine for 273¢.

36. How many pine-apples can be bought for $27.78, or 2778¢, at 6¢ apiece?

Make 27 dots, and separate them into groups of 6 each; thus,

(••••••) (••••••) (••••••) (••••••) •••

Note.—With this single line of counters before him, the pupil should now be able to answer the following questions:

1. How many such lines would represent the number of cents expressed by the first two figures of 2778¢?

2. How many dots would there be in 10 such lines? In 100? How many groups of 6 in 10 lines? In 100? How many over in 10 lines? In 100? The 300 dots over and 78 dots would make how many dots, etc.?

(From this point the process is exactly the same as in the preceding Ex. The pupil may complete the work, beginning with the making of 37 dots, if necessary, etc.)

66. The Arithmetical Solution.—6 is not contained in 2778 any *thousands* of times, for even *one* thousand times 6 would be 6000, but it is contained some hundreds of times, for 100 times 6 is only 600. The question is, *how many hundreds* of times?

In a line of 27 dots we made *4* groups of 6 dots each, in ten lines (270 dots) there would be 40 groups, and in 100 lines, 400 groups. Hence, in the arithmetical process we write *4* in hundreds' place. It shows how many *hundred* groups of 6 can be made of 27 *hundred* dots. There are 3 *hundred* remaining. These, with the 78, make 378. How many groups can be made of 378 dots?

$$6\overline{)2778}$$
$$4\,.\,.$$

In a line of 37 dots we could make 6 groups, with one dot over, and in ten lines we could make 60 groups, with 10 over. We write the *6* in tens' place. It shows how many times 10 groups of 6 each can be made of 37 *tens*.

$$6\overline{)2778}$$
$$46\,.$$

The ten over and the 8 make 18, in which 6 is contained 3 times. Hence, 6 is contained 463 times in 2778.

$$6\overline{)2778}$$
$$463$$

SLATE EXERCISES.

Find how many times

37. 4 in	**38.** 6 in	**39.** 7 in	**40.** 8 in	**41.** 9 in
296	258	656	784	399
348	462	483	608	432
264	354	598	527	873
192	408	651	770	778
356	516	619	664	617

How many times

4 in	6 in	7 in	8 in	9 in
42. 3124	**50.** 3510	**58.** 7056	**66.** 1707	**74.** 5373
43. 4796	**51.** 2578	**59.** 4606	**67.** 9608	**75.** 2961
44. 5340	**52.** 1602	**60.** 6110	**68.** 7527	**76.** 4212
45. 9516	**53.** 5958	**61.** 2072	**69.** 5768	**77.** 8136
46. 6779	**54.** 1086	**62.** 5408	**70.** 3664	**78.** 7008
47. 8040	**55.** 2802	**63.** 4260	**71.** 6856	**79.** 4905
48. 1596	**56.** 4556	**64.** 1645	**72.** 2392	**80.** 2220
49. 7120	**57.** 3062	**65.** 1113	**73.** 2216	**81.** 2052

Definitions.

67. ***Division*** is a process of finding how many times one number is contained in another, or of finding one factor of a number when the other factor is known.

Division has two uses:

1. To find how many equal parts there are in a number when we know what one part is.

2. To find one of the equal parts of a number when we know how many parts there are.

68. Terms used.—The ***Dividend*** is the number to be divided.

69. The ***Divisor*** is the number by which we divide.

70. The ***Quotient*** is the number of times the divisor is contained in the dividend.

71. A part of the dividend remaining undivided is called the ***Remainder.***

72. Signs.—There are three signs used to indicate division :

1. The divisor is placed on the right of the dividend with the sign ÷ between them ; 84 ÷ 7 is read 84 divided by 7 ; or,

2. The divisor is placed on the left of the dividend, with a curved line between them ; thus, 7)84 is read 84 divided by 7 ; or,

3. The divisor is written beneath the dividend with a horizontal line between them ; thus, $\frac{84}{7}$ is read 84 divided by 7.

Note 1.—Multiplication is taking one number as many times as there are units in another. Division is finding how many times one number is contained in another. Hence division is the reverse of multiplication.

Note 2.—One number can be taken from another as many times as it is contained in it, hence by division we find how many times one number can be subtracted from another.

Division of Numbers into Parts.

73. If a single thing or a number of things is divided into two equal parts or lots, the parts are called halves. Thus, if a boy shares an apple equally with a school-mate, he cuts it into two equal parts : each part is a half of the apple ; or, if he shares a number of chestnuts equally with another, he divides the chestnuts into two equal lots : each lot is a half of the whole number.

If a single thing or number of things is divided into three equal parts, the parts are *thirds ;* if into four equal parts, the parts are *fourths,* etc. Such equal parts are called fractions.

74. *One half* is written in figures thus : $^1/_2$; the figure above the line represents *one,* the figure below represents the word *half.*

One third is expressed by 1 above the line and 3 below it, thus, $^1/_3$; two thirds is written $^2/_3$; the 1 and the 2 represent the number of thirds, the 3 stands for the word *thirds.*

ORAL AND ILLUSTRATIVE EXERCISES.

1. A mother has a basket of pears which she wishes to divide equally among three children : what part of the whole number will she give to each one ? What part would be given to each if

they were equally divided between 2 children ? Among 5 children? 4 children ?

2. If a father divides 8 one-cent pieces among 4 children, what *part* of the number does each one receive ? How *many* is that ?

3. If he divides 18¢ among 6 children, how *many* does each one receive ? What *part* of the whole number is that ?

4. Illustrate with strips of paper, apples, or other objects, what is meant by $^1/_2$, $^1/_3$, $^2/_3$, $^1/_4$, $^2/_4$, $^3/_4$, of anything or number.

5. Show that $^2/_2$, $^3/_3$, $^4/_4$, of anything are equal to the whole of it.

6. Illustrate with objects what is meant by $^3/_4$ of 24 ; also,

$^1/_8$ of 32,
$^1/_3$ of 27,
$^1/_6$ of 54,
$^3/_7$ of 28,
$^2/_8$ of 56,
$^1/_4$ of 36,
$^5/_6$ of 24,
$^3/_8$ of 72,
$^4/_5$ of 50.

7. *Write in figures.*

four sixths,
three sixths,
one sixth,
two sixths,
six sixths,
three tenths,
five sixths,
one third,
three thirds,
two tenths,
six sevenths,
two sevenths,
four tenths,
six fourths,
three sevenths,
five sevenths,
one seventh,
seven sevenths,
six sevenths,
four eighths,
seven tenths,
five eighths,
eight ninths,
seven eighths,
five ninths,
nine tenths,
three ninths,
seven fifths,

The Two Uses of Division Compared.

75. The two uses of division which are represented in the following problems are often confounded. The figures employed in the arithmetical solutions, and the digits in the answers, are exactly the same for both, yet the answers are really different, and the explanations of the process by which they are obtained should vary accordingly.

1. How many times is 5 contained in 45 ?

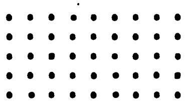
2. One fifth of 45 is how many ?

The arithmetical solution to both is the same, thus

$$45 \div 5 = 9.$$

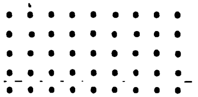
The Solution with Counters.—That the pupil may clearly understand the essential differences between the two problems, and yet why their modes of solution should be the same, let him solve them with counters, thus:

1. To find how many times 5 are contained in 45, he puts down 45 counters, arranging them in groups of 5 as he does so.



The answer to this is 9 *groups* of 5, or *9 fives.*

2. To find $^1/_5$ of 45 he does precisely the same as before, *and then takes one counter from each group*, thus getting $^1/_5$ of all the groups, or $^1/_5$ of 45.



The answer to this is *9 units.*

In this way it is shown that *there are as many units in $^1/_5$ of 45 as 5 is contained times in 45.*

Show with counters how to find

1. $^1/_3$ of 18
2. $^1/_4$ of 28
3. $^1/_6$ of 30
4. $^1/_4$ of 36
5. $^1/_6$ of 42
6. $^1/_5$ of 55
7. $^1/_6$ of 18
8. $^1/_7$ of 49
9. $^1/_8$ of 40
10. $^1/_9$ of 54
11. $^1/_8$ of 72
12. $^1/_6$ of 54

OBJECTIVE EXERCISES.

Work out the following problems with the aid of counters, without using your knowledge of the multiplication table, and in each case state whether it is your object to find how many there are in a lot; or, how many lots there are.

1. Six gentlemen on a fishing excursion catch 48 fish, and divide them equally among themselves, how many does each one receive? How many poor families would they supply if 6 fish were sent to a family?

2. A dairyman has 96 pounds of butter to be sent to market, how many jars will he need if he puts 8 lbs. in a jar? How many pounds must he pack in a jar if he has but 8 jars?

3. A lady pays $42 for 14 yards of silk, how much does she pay a yard? If instead of the silk she buy velvet at $6 a yard, how many yards would she get for the same money?

4. A teamster hauls 9 barrels of coal oil at a load; how many loads does he make of 126 barrels? He puts them into 3 freight cars; how many in a car?

ORAL AND SLATE EXERCISES.

1. Five ninths of 54 are how many?

Analysis.—$^{1}/_{9}$ of 54 is 6; and $^{5}/_{9}$ of 54 are 5 times 6 = 30.

How many are

2. $^{3}/_{8}$ of 72
$^{4}/_{9}$ of 36
$^{7}/_{8}$ of 48

3. $^{4}/_{9}$ of 54
$^{5}/_{6}$ of 42
$^{3}/_{5}$ of 45

4. $^{3}/_{4}$ of 32
$^{4}/_{9}$ of 27
$^{7}/_{8}$ of 32

5. $^{7}/_{10}$ of 90
$^{6}/_{7}$ of 28
$^{8}/_{9}$ of 81

6. $^{5}/_{6}$ of 72
$^{5}/_{9}$ of 63
$^{5}/_{8}$ of 64

7. $^{4}/_{7}$ of 63
$^{5}/_{8}$ of 56
$^{3}/_{4}$ of 32

8. $^{6}/_{7}$ of 35
$^{3}/_{8}$ of 48
$^{4}/_{5}$ of 20

9. $^{3}/_{7}$ of 49
$^{4}/_{9}$ of 72
$^{5}/_{12}$ of 84

How many are

10. $^{2}/_{5}$ of 725
11. $^{2}/_{3}$ of 891
12. $^{5}/_{6}$ of 582
13. $^{4}/_{7}$ of 861
14. $^{3}/_{8}$ of 520
15. $^{6}/_{7}$ of 8008
16. $^{5}/_{9}$ of 4392
17. $^{11}/_{12}$ of 2196
18. $^{5}/_{7}$ of 4011
19. $^{9}/_{10}$ of 7680
20. $^{4}/_{5}$ of 18365
21. $^{7}/_{8}$ of 93208
22. $^{5}/_{7}$ of 98098
23. $^{7}/_{9}$ of 35172
24. $^{2}/_{7}$ of 31738

Remainders, and how to Treat them.

76. 1. To divide seven apples equally between two persons, we would divide 6, the greatest number of them that can be so divided without cutting any, and then we would cut the remaining apple into two equal parts, and give one part to each person.

In like manner a half of 7 is obtained arithmetically by first finding how many there are in a half of 6, and adding thereto one half of the remainder.

$$\begin{array}{l} 2)\underline{7} \\ \;\;3^{1}/_{2} \end{array}$$

2. If 11 apples were to be divided equally among three children, we would divide 9, the greatest number of them that can *be so divided* without cutting, into 3 equal lots; then cutting

each of the two remaining apples into thirds, we would put two of the parts with each lot.

In like manner the $^{1}/_{3}$ of 11 is found arithmetically by finding first the $^{1}/_{3}$ of the greatest number in 11 that can be divided by 3 without a remainder, and then adding thereto $^{1}/_{3}$ of the remainder.

$$\begin{array}{r} 3\underline{)11} \\ 3\,^{2}/_{3} \end{array}$$

ILLUSTRATIVE EXERCISES.

Show by the division of two strips of paper of the same length and breadth, or by two lines drawn side by side and of the same length, that $^{2}/_{3}$ of 1 are equal to the $^{1}/_{3}$ of 2.

1. Divide 7 sticks of candy equally among 5 children. Illustrate the actual division by lines upon the slate; also perform the example arithmetically.

2. Divide 9 pencils among 4 boys; 7 yards of ribbon among 5 girls. (Illustrate as above.)

3. Divide 11 dollars among 4 persons; 13 dollars among 3 persons; 17 dollars among 5 persons. (Illustrate with counters.)

4. Divide 7 feet into 3, 9 inches into 4, 13 yards into 6, 19 feet into 7 equal parts.

5. Divide 16 min. into 5, 21 h. into 6, 39 d. into 9 equal parts.

SLATE EXERCISES.

These examples may be read thus: find $^{1}/_{3}$ of 235; $^{1}/_{4}$ of 457, etc., etc., or how many times 3 in 235.

6. 235÷3=
7. 457÷4=
8. 368÷5=
9. 279÷6=
10. 463÷7=
11. 749÷8=
12. 885÷9=
13. 453÷8=
14. 493÷3=
15. 743÷6=
16. 621÷7=
17. 349÷3=
18. 573÷4=
19. 831÷4=
20. 756÷5=
21. 492÷5=
22. 567÷8=
23. 385÷6=
24. 745÷6=
25. 946÷7=
26. 853÷7=
27. 345÷8=
28. 700÷9=
29. 600÷9=
30. 431÷8=
31. 507÷8=
32. 620÷7=
33. 470÷7=
34. 583÷6=
35. 497÷6=
36. 349÷5=
37. 463÷4=

38. 4821÷3=
39. 5678÷4=
40. 4566÷5=
41. 8321÷6=
42. 4720÷7=
43. 5008÷8=
44. 6000÷9=
45. 4725÷8=
46. 9613÷7=
47. 7895÷6=
48. 4567÷5=
49. 8765÷4=
50. 9463÷5=
51. 4407÷6=
52. 8371÷7=
53. 9462÷9=
54. 4587÷8=
55. 5349÷9=
56. 4623÷8=
57. 7000÷7=
58. 76543÷4=
59. 49021÷5=
60. 74935÷6=
61. 68427÷7=
62. 54379÷8=
63. 48628÷9=
64. 34567÷8=
65. 50021÷7=
66. 38745÷6=
67. 74960÷5=
68. 44312÷7=
69. 57368÷8=
70. 49564÷9=
71. 87310÷8=
72. 40302÷7=
73. 50000÷4=
74. 46738÷6=
75. 27493÷3=
76. 56843÷5=
77. 74219÷8=

Applications.—1. How many pounds of beef can be bought for 1854¢, at 9¢ a pound ?

2. There are 12 boys on 6 sleds; what part of the boys to each sled ? A regiment of 531 men is transported in 9 cars ; how many men to each car ? What part of the regiment ?

3. A man divides $58424 among his 8 children ; how much does each one get ? What part of the whole is that ?

4. A carpenter cuts a strip of molding 192 inches long into 8 equal pieces. How long is each piece ? Suppose he cuts it into 4 equal pieces; how long will each be ?

5. There are 5280 feet in a mile, how many in $^1/_4$ of a mile ?

6. If a boy's hoop measures just 8 feet in circumference (around it), how many times will it revolve (turn round) in a half mile ?

7. If I pay 15¢ for 3 lead-pencils, what part of the money do I pay for one ? If I pay $15759 for 9 lots, how much do they cost me apiece ? What part of the whole sum does each lot cost ?

8. If a ship sails 1918 miles in two weeks at a uniform rate of speed, what part of the distance does she sail in one week ? How many miles ? What part of the whole distance does she sail in one day ? How many miles ? What part of the whole *distance in two days ?* How many miles ?

Long Division.

77. 1. How many times is 37 contained in 9386 ?

Here we have a divisor which does not occur as a factor in the multiplication table, hence we construct a table specially for it. Having done this, we proceed with the division exactly as we do with divisors less than 10, except, 1st, that we write down the products and remainders because too large to carry in the mind ; and 2d, that we place the quotient over the dividend that it may be out of the way of the written work which is to follow.

We first find in this table that 2 times 37 = 74 ; hence we know that 37 is contained two times in 93, and therefore 2 *hundred* times in 93 *hundred.* We place the 2 in the order of hundreds (over hundreds' place in the dividend), and subtract the 74 *hundred* from the 93 *hundred,* and obtain the remainder, 19 *hundred.* Now, instead of carrying this to the 8 mentally, we annex the 8 to the 19, and thus obtain 198 tens for the second partial dividend.

Table of Multiples of 37.

1×37= 37
2×37= 74
3×37=111
4×37=148
5×37=185
6×37=222
7×37=259
8×37=296
9×37=333

Process of Division.

$$\begin{array}{r} 253^{25}/_{37} \\ \hline 37)9386 \\ 74\mathit{00} \\ \hline 198\mathit{6} \\ 185\mathit{0} \\ \hline 136 \\ 111 \\ \hline 25 \text{ Rem.} \end{array}$$

Again, by referring to our table, we find that 37 is contained in 198 (tens) 5 (tens) times, this we write in the order of tens, and, subtracting 185 from 198, we get 13 (tens) for a remainder. To this we bring down the 6 units and proceed as before.

Thus we find that in 9386, 37 is contained 253 times, with a remainder of 25. This can be proved, for 253 times 37 are 9361, and the remainder, 25, being added to 9361, the sum is 9386.

Note.—The terms of the several partial dividends that are at the right of the first figure brought down, and the ciphers annexed to the several products, may be omitted in the work, since they have no effect in the result. In the process above, the figures that may be omitted are printed in italics.

SLATE EXERCISES.

Find how many times 37 in

2. 9860
3. 7935
4. 5047
5. 8376
6. 9098
7. 5672
8. 2035
9. 8980
10. 6050
11. 5817
12. 3542
13. 9273

Note 1.—In the following exercises, 14–40, construct a table of multiples for each divisor. These exercises can be carried to any desirable extent. The divisors remaining the same, the same table of multiples will suffice for thousands of examples. It will be well to practice the pupils in this way till they are thoroughly familiar with the process of long division. They will then find little difficulty in obtaining quotient figures without the aid of tables.

Note 2.—The table of multiples may be formed by adding the divisor to itself for the second multiple, next by adding the divisor to this sum, and so on, till the tenth multiple is reached. If this be the same as the divisor, with a cipher annexed, the result is a good test of the accuracy of the table.

Find how many times

54 in	49 in	64 in	83 in	98 in
14. 872	**19.** 658	**24.** 7856	**29.** 87506	**34.** 296725
15. 953	**20.** 9030	**25.** 4785	**30.** 84378	**35.** 875682
16. 428	**21.** 720	**26.** 9378	**31.** 59643	**36.** 937865
17. 397	**22.** 692	**27.** 2704	**32.** 23232	**37.** 468728
18. 685	**23.** 5377	**28.** 1921	**33.** 12345	**38.** 321485

39–40. How many times 98 in 5764328694761? In 358674930006?

Definitions.

78. When the steps of the solution are all written, as in the preceding examples, the process is called ***Long Division.***

79. Any part of a dividend used to obtain a quotient figure is called a ***Partial Dividend.*** (It is only a part of the entire dividend. See also *partial* product.)

80. The use of the multiple tables is convenient when we have to employ the same divisor many times successively, as in the foregoing exercises, but it would involve a great deal of unnecessary labor to construct one for every divisor we may happen to have. Hence it is desirable to learn how to obtain a quotient figure without the aid of such a table. In doing this the learner should ask himself, "What must I multiply the divisor by, so as to obtain a product *not greater* than the partial dividend, and *not so small* that the remainder will be equal to or greater than the divisor?"

Note.—If the product is greater than the partial dividend, the term proposed for the quotient is too great, and if the remainder is equal to or greater than the divisor, the proposed term is too small.

SLATE EXERCISES.

1–5.
234÷11=
543÷11=
754÷21=
638÷21=
497÷31=

6–10.
536÷31=
635÷41=
874÷41=
504÷52=
970÷52=

11–15.
743÷62=
634÷72=
549÷82=
638÷53=
543÷95=

16–20.
8456÷51=
2345÷51=
3856÷51=
7432÷61=
1579÷61=

21–25.
3842÷71=
6548÷71=
7432÷81=
9465÷81=
4567÷91=

26–30.
8461÷82=
7111÷73=
9000÷64=
4050÷55=
6031÷46=

31–35.
234÷19=
765÷24=
801÷35=
743÷46=
257÷58=

36–40.
684÷69=
943÷13=
976÷25=
564÷36=
310÷47=

41–45.
240÷58=
589÷69=
432÷88=
345÷77=
678÷59=

46–50.
3654÷57=
7890÷65=
2345÷78=
7937÷47=
2468÷38=

51–55.
3579÷54=
1357÷29=
4682÷37=
7038÷76=
4925÷89=

56–60.
6492÷88=
7483÷73=
6294÷97=
7385÷68=
4291÷51=

61–65.
406÷23=
709÷34=
305÷54=
407÷56=
808÷65=

66–70.
500÷74=
400÷83=
300÷92=
500÷94=
800÷52=

71–75.
4000÷32=
2000÷43=
6000÷54=
3000÷65=
6000÷76=

76–80.
4000÷87=
3000÷96=
4000÷65=
7000÷44=
9000÷33=

Find how many times

81. 326 in 1630
82. 251 in 2362
83. 347 in 1829
84. 628 in 2654
85. 592 in 1867
86. 489 in 4375
87. 981 in 982
88. 873 in 7756
89. 784 in 7830
90. 892 in 8000
91. 384 in 684
92. 721 in 1223
93. 876 in 1676
94. 988 in 9875
95. 876 in 8759

96. 428 in 12415
97. 326 in 24081
98. 234 in 13462
99. 435 in 15781
100. 1723 in 344680
101. 2938 in 357264
102. 9321 in 993280
103. 8746 in 785463
104. 5932 in 593175

General Rule for Division.

81. *Rule.*—**1. Write the divisor at the left of the dividend with a right curve between them.**

2. For the first partial dividend take only as many figures at the left of the given dividend as would, if considered apart from the rest, express a number great enough to contain the divisor.

3. Find the greatest number by which you can multiply the divisor to make a product not greater than this partial dividend, and place it in the quotient.

4. Multiply the divisor by this number, subtract the product from the partial dividend, and to the remainder annex the next figure of the dividend. If the result is equal to or greater than the divisor, it is the second partial dividend, but if less, continue to annex figures from the dividend in their order, placing a cipher in the quotient for each figure brought down, till a partial dividend is formed; or, till all the figures of the dividend have been brought down.

5. Proceed with the second partial dividend as with the first, and so on, till all the terms of the dividend have been used. The result will be the quotient sought.

6. If, after the last division, there be a remainder, place it with the divisor underneath, at the right hand of the quotient.

Proof.—**Multiply the divisor by the quotient, and to the product add the remainder, if any. The result will be equal to the dividend if the work is correct.**

Note.—The learner should write each term of the quotient over the last figure of the dividend from which it was obtained. It will save him from some mistakes *to which he is liable.*

Applications.—1. Distribute $13425 equally among 27 sailors. How much will each one receive?

2. If a locomotive can run 513 miles in 19 hours, how far can it run in one hour? In two hours? In 10 hours?

3. The steamer Suevia makes the trip from New York to Hamburg in 12 days. The distance is 3408 miles. How many miles per day does she make? How many in 6 days?

4. How many pounds are there in 352 ounces?

5. How many days in 3567 hours? In as many minutes?

6. How many hours in 4628 minutes? How many days?

7. If 20 horses eat 1940 bushels of oats in a year, how many will one horse eat in the same time?

8. If one boy picks a barrel of apples in an hour, what part of the time ought it to take two boys to do the same work? Three boys? If one man can dig a ditch in 54 hours, how long will it take 9 men to dig it?

9. What number multiplied by 23 will give 36087?

10. How many times can 27 be subtracted from 62397?

11. If the product of two factors is 21015, and one factor is 45, what is the other factor?

12. When potatoes are 75¢ a bushel, how many bushels can I purchase for 675¢?

13. A quire of paper has 24 sheets. How many quires are there in 5631 sheets? In 1436 sheets?

14. What is the price of a barrel of apples, if 36 barrels cost $90?

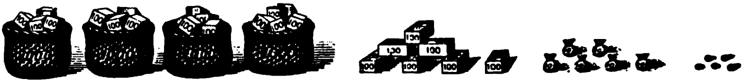
15. At a post-office there are 812 boxes in 14 rows, how many are there in a row?

16. If you weigh 1476 ounces, how many pounds do you weigh? How many pounds does your sister weigh, her weight being 133 ounces less than yours?

17. Bought 897 acres of land for $77142. How much did I pay per acre?

When the Divisor has One or more O's at the Right.

82. A boy employed at a toy-shop had to put a large number of marbles into little canvas bags, which were to be sold with the marbles. He put ten marbles into a bag, and when he had thus filled ten bags, he put them into boxes, and ten of these boxes he put into a basket to be taken to the store-room. When the work was done there were



Express the number of marbles in figures.

ILLUSTRATIVE AND SLATE EXERCISES.

1. *a.* How many *baskets* full in the lot of marbles above represented, and what would be left if they were taken away? How many marbles would remain?

b. How many thousands in 4765, and how many over?

Arithmetical Process.

1|000)4|765
4 — 765 Rem.

c. What do you notice on comparing the figures of the quotient and remainder with the figures in the dividend?

2. *a.* How many times 2 baskets full in the lot, and how many would remain if two times 2 baskets full were taken away?

b. How many times 2000 in 4765, and how many over?

Arithmetical Process.

2000)4765(2 Or, 2|000)4|765
4000 2 — 765
765

Note.—The *quotient* figure is the same as if 4 were divided by 2. The figures of the *remainder* are the same as the 3 right-hand figures of the dividend, which stand for the boxes, bags, and single marbles that would be left if 2 times 2 baskets *were taken away.*

3. *a.* How many times 3 baskets full in the lot, and what would be left if they were taken away?

b. How many times 3000 in 4765, and how many over?

Arithmetical Process.

$$\begin{array}{l} 3000)4765(1 \\ \quad\;\; \underline{3000} \\ \quad\;\; 1765 \text{ Rem.} \end{array} \qquad \text{Or,} \qquad \begin{array}{l} 3|000)\underline{4|765} \\ \qquad\quad 1\text{—}1765 \text{ Rem.} \end{array}$$

Note.—Here it will be noticed that the result is the same as if 4 were divided by 3 and the remainder prefixed to the figures cut off from the dividend.

4. *a.* How many boxes in the lot, including those in the baskets, and the single boxes represented?

b. How many times 23 boxes in the lot, and what would be left if 2 times 23 boxes were taken away?

c. How many times 2300 in 4765, and how many remaining?

Arithmetical Process.

$$\begin{array}{l} 2300)4765(2 \\ \quad\;\; \underline{4600} \\ \quad\;\;\; 165 \text{ Rem.} \end{array} \qquad \text{Or,} \qquad \begin{array}{l} 23|00)47|65(2 \\ \qquad\quad \underline{46} \\ \qquad\quad\; 165 \text{ Rem.} \end{array}$$

Note.—The result is the same as if 47 were divided by 23, and the remainder prefixed to the figures cut off from the dividend.

Hence, to shorten the work of division when the lower orders of the divisor are filled with ciphers, we have the following

83. ***Rule.*****—Cut off the 0's of the divisor, and as many figures at the right of the dividend. Divide as if the parts left were the entire divisor and dividend. The remainder, if any, and the figures cut off from the dividend, form the true remainder.**

SLATE EXERCISES.

1. 567÷40=
2. 876÷50=
3. 393÷60=
4. 584÷70=
5. 748÷80=
6. 569÷90=
7. 4478÷80=
8. 2345÷60=
9. 6789÷70=
10. 3456÷80=
11. 7482÷90=
12. 3925÷90=
13. 6783÷80=
14. 4571÷70=
15. 78351÷20=
16. 46228÷30=
17. 57135÷40=
18. 46287÷70=
19. 34541÷80=
20. 26483÷90=
21. 98765÷80=
22. 123456÷70=
23. 700123÷60=
24. 845679÷50=

Shorter Method of Computing in Long Division.

84. Many rapid accountants dispense with written products in long division. They form the remainder by writing down at once what the several terms of the product lack to make up the partial dividend.

Example.—1. How many times 956 in 3681 ?

3
956)3681
2868
813

Explanation.—The work at the left exhibits the steps of the operation as already learned. How the written product may be dispensed with is shown in the work at the right, for which the following wording is a sufficient explanation.

3
956)3681
813

Wording.—18 and ***3*** are 21 (carry 2); 15, 17, and ***1*** are 18 (carry 1) ; 27, 28, and ***8*** are 36. Don't *say* carry 2, etc., but *do it.*

The numbers in ***heavy italics,*** occurring after the word "and," are written while they are being pronounced.

2. How many times is 217 contained in 7507083 ?

Written Work.

$34594\frac{185}{217}$
217)7507083
997
1290
2058
1053
185

Explanation.—217 being contained 3 times in 750, we multiply, and write down what the product lacks to make up 750.

Wording.—21 and ***9*** are 30, carry 3 ; 3, 6, and ***9*** are 15, carry 1 ; 6, 7.

Annexing the next figure of the dividend, we have 997 for the second partial dividend ; 217 being contained in 997 four times, we set 4 in the quotient, and proceed as before.

Wording.—28 and ***9*** are 37, carry 3 ; 4, 7, and ***2*** are 9 ; 8 and ***1*** are 9.

For Practice in the Shorter Method.

How many times

3. 72 in 856
4. 83 in 984
5. 87 in 899
6. 56 in 934
7. 111 in 5935
8. 222 in 7356
9. 333 in 6931
10. 235 in 9871
11. 354 in 98763

SLATE EXERCISES.

1. 45600÷100=
2. 72400÷300=
3. 45600÷500=
4. 83600÷700=
5. 73500÷900=
6. 47400÷400=
7. 43000÷1000=
8. 234000÷4000=
9. 645000÷6000=
10. 840000÷8000=
11. 375000÷3000=
12. 4687000÷5000=
13. 99120÷ 590=
14. 85800÷ 780=
15. 207900÷ 630=
16. 10800÷ 270=
17. 2090000÷3800=
18. 1617000÷4900=

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19. 4276÷201=
20. 5318÷102=
21. 3725÷305=
22. 4943÷406=
23. 3756÷507=
24. 35312÷342=
25. 44325÷429=
26. 73812÷368=
27. 44831÷496=
28. 34052÷504=
29. 365212÷7040=
30. 456721÷8050=
31. 843956÷9002=
32. 433421÷6302=
33. 346549÷5900=

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34. 87654÷743=
35. 94615÷685=
36. 34641÷567=
37. 64925÷784=
38. 36287÷1926=
39. 40032÷1835=
40. 50607÷1749=
41. 48325÷1683=
42. 346819÷4297=
43. 543726÷7453=
44. 492570÷6853=
45. 749256÷9469=

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46. 3654701÷4372=
47. 2043217÷6482=
48. 4700081÷6395=
49. 6127421÷9362=
50. 5432101÷7408=
51. 4382146÷8432=
52. 7040047÷9069=
53. 2468301÷7456=
54. 7654321÷6435=
55. 5043062÷4372=
56. 3489719÷9348=
57. 7154327÷8745=

Self-Testing Problems.

Note.—Divide each dividend by all the divisors. The remainder, if any, will, in each example, be divisible by 9.

1. 53146827 ÷549
2. 61327548 ÷558
3. 128761353÷567
4. 123456789÷576
5. 987654321÷585
6. 963187452÷594
7. 712345689÷711
8. 723918645÷729
9. 791352468÷738
10. 356912478÷747
11. 981762345÷756
12. 765432189÷765
13. 781965423÷774
14. 783993456÷783
15. 792345681÷792
16. 829713456÷828
17. 846123579÷ 846
18. 864123597÷ 864
19. 709005474÷ 882
20. 470049570÷ 918
21. 357114636÷ 936
22. 987654321÷ 954
23. 976548321÷ 972
24. 981234567÷ 981

Applications.—1. If a clock ticks 29,347,095 times in a year of 365 days, how many times does it tick in a day?

2. Divide nine million nine hundred ninety-eight thousand five hundred fifty-seven by seven thousand eight hundred forty-two, and write out the answer in words.

3. The Valley Railroad is 271 miles long, and cost $5,272,305. What was the cost per mile?

4. If a farmer had 138 acres in wheat, from which he harvested 3692 bushels, how many bushels did he raise per acre?

5. A milliner cuts 7 pieces of ribbon, each 10 yards long, into pieces each 27 in. long. How many such pieces has she, and how many and how long are the waste pieces?

6. In a week a boy gathers 192 bushels of apples; how many bushels does he average per day?

7. A farmer raises 1875 bushels of wheat, which he exchanges for flour, at the rate of 5 bushels of wheat for one barrel of flour. How much flour does he receive?

8. Find how many gallons of milk in 8 cans that hold respectively 92, 102, 170, 89, 97, 125, 106, and 56 pints?

9. Suppose that two cans of equal size together hold 376 pints; how many gallons are there in each?

10. How many poor families may be supplied from 37 barrels of flour, allowing 28 pounds to each, a barrel of flour weighing 196 pounds?

11. If you take 86 steps in a minute, how many hours and minutes will it take you to walk 38,270 steps?

12. A train of 28 cars carries 493,920 pounds of flour in barrels, each barrel containing 196 pounds. How many barrels to each car?

13. In 3 weeks, a certain oil-well is said to have produced 35,000 barrels of oil. How much was that per day?

14. How many thousand make one million?

Original Problems.

Note to the Teacher.—Let the yard-stick and foot-rule be as freely used as the circumstances of the school will allow. If the foot or yard measure does not "come out even," let dimensions be given in inches, but let no account be taken of the fractions of an inch at present. No pupil should be allowed to give a problem which he has not solved himself, and the answer to which he does not know.

1. Suppose yourself to be building a pile of cubic blocks, each measuring one inch, foot, or yard on each side, the pile to be as many inches, feet, or yards, on each side, as you please, and ask to know how high you can build it with a given number of blocks.

2. If you see an oil-cloth or carpet with square figures covering a floor, measure one square, count the number on the side and end of the room, give the facts to the class, and ask them to find how long and wide the floor is.

3. Ask questions similar to these: How many states of the size of Delaware might be made out of the state of Georgia? How many cities of the population of Albany (N. Y.) might be made of the city of New York? The members of the class should hunt up the facts for themselves.

4. Give the height of one step of a stair-way, and the distance from the first to the second *floor*, second to third, etc., in some house just building, and ask how many steps will be needed.

5. State the cost of any number of things, as yards of cloth, horses, etc., etc., and the price of one to find the number. State the cost and the number, and ask the price of each.

6. Construct questions about changing hours to weeks, equal parts to wholes, etc.

7. How many days sail from —— to —— for a vessel which runs — miles per day? How many hours run for a railway train from —— to ——, at — miles per hour? (Find distances from your text book in geography, and rates of sailing from your friends.)

8. A railway train goes from —— to —— in — hours. How many miles an hour?

Note.—The newspapers often suggest interesting problems.

Principles of Division.

85. A convenient number of counters being equally distributed among 6 or 8 members of a class, let the following questions be proposed :

1. If there were twice as many counters equally distributed among the same pupils, how would each one's share be changed? If there were only half as many counters?

2. If the same number of counters had been distributed among twice as many members of the class, how would the share of each be changed? If among half as many?

3. If twice the number of counters had been given to twice as many members of the class, how would the share of each be changed? If one-half as many had been given to half as many persons?

1. How does it affect the value of a quotient to multiply the dividend by 2, by 3, by any number, while the divisor remains unchanged? To divide the dividend by 2, by 3, etc.

2. How does it affect the value of a quotient to multiply the divisor by 2, by 3, by any number, while the dividend remains unchanged? To divide the divisor by 2, by 3, etc.

3. How does it affect the value of a quotient to multiply divisor and dividend by the same number? To divide both divisor and dividend by the same number?

ORAL EXERCISES

1. How many 13's in 78? In 5×78? In $^1/_2$ of 78? In 9×78? In $^1/_3$ of 78?

2. How many times is 18 contained in 360? $^1/_2$ of 18 in 360? $^1/_6$ of 18 in 360? 4×18 in 360? $^1/_9$ of 18 in 360?

3. How many times 24 in 96? 5×24 in 5×96? $^1/_2$ of 24 in $^1/_2$ of 96? 24×24 in 24×96?

4. Divide 224 by 28; 224 by 2×28; 224 by $^1/_4$ of 28; $^1/_2$ of 224 by 28; 3×224 by 3×28; $^1/_7$ of 224 by $^1/_7$ of 28; 13×224 by 13×28.

SLATE EXERCISES.

How many times

1. 3119 in 1197696

2. 24×3119 in 1197696

3. 3119 in 1197696÷24

4. 4316 in 1031524

5. 4316÷52 in 1031524

6. 4316 in 1031524×52

7. 4316 in 1031524÷52

8. 4316×52 in 1031524

9. 4316×718 in 1031524×718

10. 4316÷52 in 1031524÷52

Division by Composite Numbers.

86. Let the pupil show, by various examples, that division by a composite number (product of two or more factors) may be performed by dividing successively by its factors. Thus, that

$$\begin{array}{r|l} 7 & 756 \\ 9 & 108 \\ \cline{2-2} & 12 \end{array} \quad \text{is equivalent to} \quad \begin{array}{l} 63)756(12 \\ \underline{63} \\ 126 \\ 126 \end{array}$$

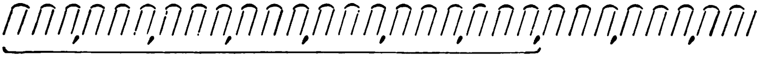
Divide in both ways

1. 18576 by 48
2. 37656 by 72
3. 30375 by 81
4. 24678 by 54
5. 391272 by 56
6. 629937 by 63

Why the results of the two methods should be the same, and how to deal with remainders when they occur in the division by factors, is shown in the solution of the following example.

7. Divide 59 by 42.

Solution with Counters.—In 59 counters there are 29 twos and 1 counter remaining; in 29 twos there are 9 sets of 3 twos and 2 twos over; in 9 sets of 3 twos each there are 1 group of 7 sets and 2 sets of 3 twos remaining; all of which is shown as follows.



Arithmetical Process.

$$\begin{array}{r|l} 2 & 59 \\ 3 & \overline{29}\text{—}1 \ldots\ldots\ldots\ 1 \\ 7 & \overline{\ 9}\text{—}2\times2=4 \ldots\ 4 \\ \cline{2-2} & \ \ 1\text{—}2\times3\times2=12 \\ & \text{Remainder } \overline{17} \end{array}$$

Explanation.—As may be seen in the illustration, the first divisor is 2 units and the remainder is a *unit;* the second remainder is *2 groups* of 2 each, and the third is *2 larger groups* of 3 twos each. The sum of these remainders is 17, the same as that obtained by dividing 59 at once by 42. Thus we obtain the rule for dividing any number by the factors of composite divisors.

87. ***Rule.*—1. Divide the dividend by any factor of the divisor, divide the quotient by another factor, and so on, till an entire set of factors has been employed.**

2. If remainders occur, multiply each by all the divisors preceding the one that produced it. The sum of the products, added to the remainder, if any, resulting from the first division, will be the true remainder.

Self-Testing Exercises.

To be Constructed by the Teacher for Dictation.

Addition and Subtraction.—**1.** Write any set of numbers, each of which shall be greater than the preceding, as, for example,

83, 237, 250, 592, 728, 851, 9872, 18589.

Subtract the first from the second, the second from the third, etc. To the sum of the remainders add the first number. If the work is correct, the sum will equal the last number.

Multiplication.—**2.** Take any set of numbers the sum of which is 10, 100, 1000, etc., multiply each by any given number, and add together the products. (See examples 7–18, p. 64.)

Division.—**3.** Take *17* and *19*. Annex *4* ciphers to each. Divide each number thus formed by the sum of *17* and *19*. If the sum of the quotients, including fractions, be expressed by 1, with *4* ciphers annexed, the work is correct.

4. Separate any number expressed by 1 with *4* ciphers annexed into any two parts, each represented by *4* figures. Take any two convenient smaller numbers, as *29* and *38*. Prefix the *29* to either of the former numbers, and the *38* to the other; thus,

*29*3276 and *38*6724 or *38*3276 and *29*6724.

Divide both numbers of either pair by ***68***, that is, the sum of *29* and *38* *increased* by 1. The test of accuracy is the same as in **3.**

The last method being somewhat complicated, the following additional example is given. We divide by the shorter method for the sake of space.

Explanation.—We take 78314 and 21686 as the two parts of 100000, and prefix 43 to one and 65 to the other. Then we divide both by the sum of 43 + 65 + 1.

$$\begin{array}{r} 40168\,{}^{2}/_{109} \\ 109\overline{)43,78314} \\ \underline{183} \\ \underline{741} \\ \underline{874} \\ 2 \end{array} \qquad \begin{array}{r} 59831\,{}^{107}/_{109} \\ 109\overline{)65,21686} \\ \underline{1071} \\ \underline{906} \\ \underline{348} \\ \underline{216} \\ 107 \end{array}$$

$$\begin{array}{r} 40168\,{}^{2}/_{109} \\ \underline{59831\,{}^{107}/_{109}} \\ 100000 \end{array}$$

(The teacher adds the quotients.)

Note.—In **3** and **4**, other numbers may be substituted for those in italics.

CHAPTER VI.

MISCELLANEOUS EXAMPLES.

Addition, Subtraction, Multiplication, and Division.

1. George Washington was born in 1732, and was 67 years old when he died; what was the year of his death? Abraham Lincoln was born 77 years later than Washington; when was he born? President Lincoln lived 56 years; in what year was he killed?

2. In what number is 244 contained 28 times?

3. How many strokes does the hammer of a clock make from 1 till 12 o'clock, if it strikes only the hours? How many in a day?

4. A man died leaving $5200 to his wife and three children. The widow received $2500, and the children shared the rest equally. How much did each one receive?

5. A dealer proposes to ship 100000 eggs in boxes containing 40 dozen each. How many boxes will he require?

6. One hundred and thirty-eight boxes of equal capacity contain 76176 eggs. How many dozen eggs in each box?

7. If I pay 45¢ for lead-pencils, at 3¢ each, how many pencils do I buy? How many if I pay 5¢ each?

8. A drover has $150. How many cows can he buy at $50 each? $25 each? How many could he buy at $45 each, and how much would he have left?

9. A son is born when his father is 33 years old. When the father is 36 years old, how *much* older is he than the son? How many times as old?

10. Twenty-four sheets of paper make a quire. How many quires in 1824 sheets? In $^1/_2$ as many sheets? In $^1/_4$ as many? In 3 times as many? In 5 times as many?

11. How many hours are there in 9480 minutes? In twice as many minutes? (Always find your answer in the shortest and most convenient way.)

12. How many days are there in 14088 hours? In ten times 14088 hours?

13. Out of 796 logs 3980 planks were sawed. How many planks were cut from each log, supposing them to have been of equal size?

14. Four boys agreed to sweep a school-house two weeks for $24, but at the end of the first week, three of them gave it up, and left the remaining boy to complete the work. How much should the last boy receive? How much each of the others?

15. The managers of an orphan asylum spend $239 per year for each child. The expenses one year were $7170. How many orphans in the asylum that year?

16. The manager of a concert sold 534 tickets at $1 apiece, 936 tickets at $2 apiece, and 257 at $3 apiece. He gave out 34 free tickets. The hall cost him $120 rent, and for gas and fuel he paid $19 extra. How much was left after all expenses were paid, including $2100 for the performers?

17. A congregation intends to build a church, which is to cost $12000. The collections already made are $524, $726, $837, $632, $439. How much is lacking?

18. Mr. Brown earns $28 while Mr. Black earns $15. How much will Mr. Brown earn while Mr. Black earns $105?

19. Express in words the product of the sum and difference of 8765 and 5678.

20. A train of 9 cars has in each car 63 passengers, of whom 4 are children. How many passengers altogether, how many adults, and how many children?

21. January 4th, paid into savings bank, $14; February 1st, paid in $13; February 28th, drew out $11; March 14, paid in $19; March 31st, drew out $25; April 24th, paid in $17; May 3d, paid in $9; May 25th, drew out $15; June 1st, paid in $16. How much had I then in bank?

22. A number of boys in a work-shop earn $7 each per week, and an equal number of younger ones earn $5 each per week. How many boys are there if their wages amount to $132 per week?

Suggestion.—Suppose they work in pairs, an older and a younger boy in a pair, how much would a pair receive? How many pairs?

23. How many times can a 5 gal. pail be filled from a cask containing 150 gal.? How many times from a cistern holding 12 such casks? 24 casks?

24. On Tuesday the Opera was attended by 2486 persons; on Wednesday by 3574 persons. How much more money received on the second day than on the first, at $2 per ticket?

25. Which is the greater, and by how much, one fifteenth of 645, or one sixteenth of 992?

26. If a man takes 92 steps in a minute, how far will he walk in 3 hours if he advances 5 feet in taking 2 steps? At the same rate, how far would he travel in 2 days of 9 hours each?

27. A railroad conductor makes two trips every day (except Sunday) from Philadelphia to New York and back. If these cities are 90 miles apart, what distance does he travel in a week? In a common year having 53 Sundays, if he has two weeks vacation?

28. How many times will a cart-wheel, 16 feet in circumference, revolve in going a mile (5280 feet)?

29. A drover paid $780 for cows and sheep. Of this sum he gave $360 for 9 cows. If a cow cost 8 times as much as a sheep, how many sheep did he buy?

30. How many feet of telegraph wire are needed to connect two stations with a double line, the stations being 37845 yards apart? How many poles would be required if set 45 yards apart?

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31. A healthy child's pulse beats 78 times a minute. How often does it beat in an hour? In 4 hours? In 8 hours?

32. Find the smallest number that must be subtracted from 9904, to leave a number that can be divided by 173 without remainder.

33. There are 35 regiments in an army, averaging 693 men in each; how many men in the army?

34. If a man earns $3 a day, how many weeks will it take him, working 6 days a week, to earn $567? $681?

35. A man bought a piece of land for $1564; he built a house on it for $642, a barn for $273, and made other improvements costing $148. He then sold it for $3000. How much did he gain?

36. A railroad 270 miles long has a station every 10 miles; how many stations has it? (There must be a station at both ends of a road.)

37. What is the length of a railroad that has 18 stations, at an average distance of 17 miles apart?

38. A family uses 7¢ worth of milk a day. What was the cost of the milk used the last 4 months of the year? (See 27, p. 49.)

39. If a man pays 7¢ a year for the use of a dollar, how much does he pay for the use of $5 for one year? Of $10, $50, $90, $200, $750? How much for each for a half year?

40. If a man pays 6¢ for the use of a dollar for one year, how much does he pay for the use of $16 for one year? How much for 5, 7, 10 years?

41. If a man pays 8¢ for the use of one dollar for a year, how many dollars can he get the use of, for a year, for 24¢, 32¢, 44¢, 56¢, 68¢, 74¢?

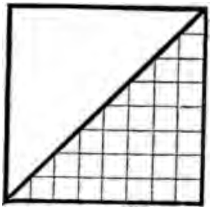
42. Sixteen messenger boys are employed to carry telegraphic dispatches, and receive 2¢ for each dispatch carried. How much does a boy earn in a week who carries 45 dispatches per day? How much do the 16 boys earn in a week if each one averages 38 dispatches per day?

43. The distance from New York city to the Cape of Good Hope is 6670 miles. When a steamer from New York is 957 miles on its way, and a steamer from the Cape is 829 miles on its way, how far apart are they, if both are on the direct line between the ports?

44. A farm has 5 fields, the 1st containing 89 acres, the 2d 101, the 3d 174, the 4th 92, and the 5th 72 acres. If the farm be re-divided into five fields of equal size, how many acres will each contain?

45. If two boys together have 9 apples, and one has 3 more than the other, how many has each? Two candidates together receive 3579 votes. How many has each, if one is elected by a majority of 291?

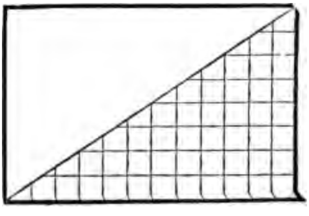
46. If a square sheet of paper be cut, from corner to corner, into two equal parts, we shall have two three-sided pieces, called ***Triangles,*** each of which is one half of the square. From this hint can you reckon how many square inches there are in a triangle such as the one at the left, if the base (the side on which it stands) measures 8 in. and the height 8 in.?



47. How many square inches would such a triangle contain if the base and height were each 87 in.?

48. If the above figure represents a triangular piece of ground, the base of which is 42 ft., and the height the same, how many square feet does the lot contain?

49. Now, suppose we had such a triangle as the one at the right; how many square inches would it contain if the base were 12 inches and the height 8 inches?



50. How many square yards does a triangle contain, the base being 672 yards, and the height 84 yards?

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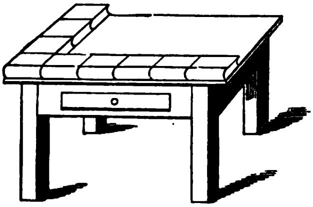
51. Suppose that you can lay 4 and 6 books along the edges of a table, as in this engraving; how many books can you lay on the table in one layer covering the top? How many times 6 books? How many times 4 books?



52. How many could you place on the table in 2 layers, 5, 7, 10, 12, 16, 24 layers?

53. If you could lay 9 books end to end along the side, and the width of the table were 5 times the width of the book, how many books could you put on the table in 1 layer, 7, 9, 25 layers?

54. How many books can you place on a table that is twice the length and three times the width of a book, if you make the pile 15 books high?

55. How many books can you put in a pile 9 books long, 7 books wide, and 31 books high?

56. How many square blocks, 6 inches long and wide, can be piled on a platform 72 inches long and 48 inches wide, if the pile is made 30 blocks high?

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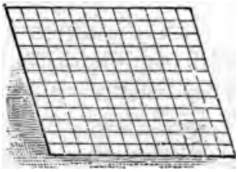
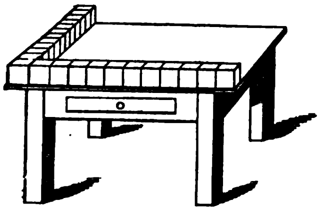
57. This is the picture of a square board divided by lines into small squares, each supposed to be 1 inch long and 1 inch wide. How many inches long is the board? How many inches wide? Count. How many squares on the upper edge? How many rows of squares? How many squares on the whole board?



58. How many square yards in a lot 23 yards long and 10 yards wide? In one 17 yards long and 13 yards wide?

59. How many square yards in a lot 20 yards wide and 30 yards long? In one 180 feet long and 270 feet wide? **(How many yards in 180 feet? In 270 feet?)**

60. If the blocks represented in the engraving are an inch long, an inch wide, how many inches long and wide is the table? How many such blocks can you place in 1 layer if you cover the top of the table?



61. How many in 5, 7, 12, 9, 13, 4, 15, 6 layers?

62. Suppose the table to be 3 feet long and 3 feet wide, how many blocks, a foot long and a foot wide, can you place exactly on the front and left edges of the table? How many would exactly cover the top? How many feet high would you have to make the pile of blocks to make it as high as it is long and wide? How many blocks, each a foot high, would there be in such a pile?

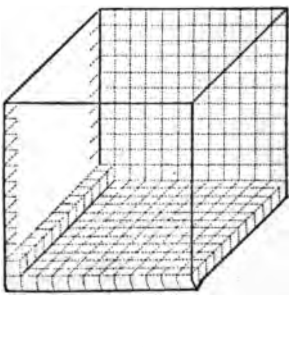
63. Could you tell without counting, how many blocks there would be in $^1/_3$ of the pile? $^2/_3$?—How many blocks would there be in $^1/_3$ of one layer? $^2/_3$ of one layer? $^1/_3$ of 2 layers? $^2/_3$ of 2 layers?

64. If with blocks, each 1 foot long, 1 foot wide, and 1 foot high, you make a pile 12 feet long, 10 feet wide, and 6 feet high, how many blocks will there be in one layer? How many in the pile?

65. How many blocks, each an inch long, wide, and high, can be placed in a box measuring on the inside 12 inches in length, width, and depth? If it measures 24 inches each way? 36 inches each way?

66. How many cubic blocks, measuring a foot each way, can be sawed out of a block of stone measuring a yard each way?

Note.—A block measuring a foot long, a foot wide, and a foot high, is called a *cubic foot*. Cut a *cubic inch* out of a piece of wood, or make one of clay. Each side of a cubic inch is a *square inch*. Each edge is an inch long.



67. How many blocks, measuring 1 inch each way, can be cut out of a block of clay measuring 12 inches each way?

68. Find how many cubic feet in a block of marble 5 feet long, 3 feet wide, and 2 feet thick.

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69. A swallow flies on an average 2640 yards per minute. How many miles does it fly in 4 hours?

70. A pipe discharging 9 qt. of water every minute fills a reservoir in 4 hours. How many gallons does the reservoir hold?

71. A milkman brought 12 cans of milk into town. Three of these contained 8 gallons each, 5 contained 18 gallons each, and the others together contained 47 gallons. How many pints of milk did he bring to town?

72. A gentleman once said: "If I had saved only 5¢ a day, since I was 20 years old, my savings would now amount to $730." How many years old was he? (How many cents in $730?)

73. A mill-wheel makes 4 rotations in a minute. How many hours must it revolve to make 1800 rotations?

74. In the center of a room, 12 feet square, lies a rug 6 feet long and 5 feet wide. How many square feet of the floor is uncovered? (Draw a diagram.)

75. A garden, in the form of a rectangle, is 72 yards long and 50 yards wide. In it stands a green-house 22 feet long and 12 feet wide, and a summer-house 10 feet long and 10 feet wide. How much space of the garden is not under cover?

76. Mr. Southworth received 15 boxes of tea, the boxes and the tea together weighing 570 lb. How much did the tea in each box weigh, if the empty boxes weighed each 3 lb.?

77. $2000 is to be distributed among 3 persons. Mr. A. is to receive $^1/_5$ of it, Mr. B. $^1/_4$, and Mr. C. the remainder. How much *will each one* receive?

78. A certain book contains 35220 lines. How many days will it take you to read it through, at the rate of 587 lines a day?

79. If I give 153 barrels of flour, worth $6 a barrel, in exchange for 54 acres of land, how much do I pay per acre for the land?

80. In a certain year Mississippi produced 997,576 bales of cotton (450 lbs. to the bale). What was its value at 11¢ a pound?

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81. Four boys at a picnic make the following inventory (list) of their property:

William	has	10¢,	20	marbles,	5	arrows,	6	crackers,	4	apples.
Monroe	"	18¢,	16	"	3	"	5	"	2	"
Harry	"	6¢,	50	"	9	"	7	"	2	"
James	"	22¢,	18	"	7	"	2	"	4	"

Find the average number of cents, the average number of marbles, the average number of arrows, etc., which they have; or, in other words, find how much money each would have if the property were equally divided among them. How many marbles, arrows, etc.?

82. The same boys find the weight of each to be as follows: William's, 85 lb.; Monroe's, 78; Harry's, 86; James's, 92. What is their average weight?

83. A butcher buys hogs weighing severally 268, 372, 283, 356, 289, 316, 328 lb. What is their average weight?

84. What is the average value of the hogs at 4¢ per pound? What is the value of the 7 hogs at the rate thus found? Find the value of the hogs separately, and add them together.

85. Four boys wish to find the average age of their fathers. William's father is 52; Monroe's, 48; Harry's, 45; and James, who has made the calculation, says that his father's age makes the average age of the four just 49 years. How old was James's father?

86. The population of Cleveland was 92,829 in 1870; ten years later it was 160,146. What was the average increase per year?

87. Bought 5 yd. of silk, at $2 per yd.; 13 shirts, at $2 apiece; 6 pairs of socks, at $0.50 a pair; and gave the merchant a $50 bill. What change did I get?

88. One omnibus contains 23 persons, another 32, and a third 26. If two persons leave one of them, and 11 are taken up on the way, how can the party be so divided that the omnibuses shall hold equal numbers?

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89. Add 352, 6324, 497, and 723; subtract 3647 from the sum; multiply the remainder by 84, and divide the product by 114. What is the quotient?

90. Add 2839, 44051, 6273, 78495, 9617; multiply the sum by 27, and from the product subtract the sum of the following numbers: 1827, 3929, 8272, 13764.

91. How much is $^1/_2$ of 96?—$^1/_4$ of 96?—$^1/_8$ of 96? How much is $^3/_4$ of 9856?—$^5/_8$ of 7656?—$^8/_9$ of 3618?

92. From 1000 subtract 367. Multiply 582 by the subtrahend, also by the remainder. Add together the products. Can you find what the sum should be without performing the work in full?

93. Divide the sum of the products of (56 × 37) and (44 × 37) by 100, and tell why the digits of the answer should be the same as those of the multiplicand.

94. The great bridge from New York to Brooklyn is suspended from 4 cables, each composed of 6300 wires, each 3578 feet long. How many feet would all these wires extend if laid end to end? How many yards? How many miles? (5280 feet = 1 mile.)

95. If laid end to end, how many times would the wires of the bridge extend from New York to Philadelphia (90 miles)? From New York to Chicago (980 miles)? From New York to San Francisco (3400)? (Work in miles, reject remainders.)

96. In 90 years the total population of the United States increased from 3,929,214 to 50,155,783. What was the average increase per year? Per decade? (A decade is 10 years.)

CHAPTER VII.

UNITED STATES MONEY.

88. The money of the United States consists of coins made at the United States mints, and government or bank notes duly authorized by law.

Coin is the specie or metallic currency, and notes are the paper currency, of the country. (Compare the words current and currency.)

The pupils should be required to write for themselves a list of the coins used, being left to get the information as they can.

89. The following are the denominations of money used in business and in accounts :

10 Mills = 1 Cent, 100 Cents = 1 Dollar.

This table is the correct business form, but the decimal character of our currency is better shown in the following table:

10 Mills = 1 Cent, 10 Cents = 1 Dime, 10 Dimes = 1 Dollar.

The word *mill* is from the Latin *mille, a thousand* (1000 m. = $1). *Cent* is from the Latin *centum, a hundred* (100¢ = $1). *Dime* is from the old French word *disme, ten* (10 dimes = $1).

90. The sign for dollars is $; parts of a dollar are indicated by a point called a separatrix. Both signs are prefixed to the figures to which they belong.

Dollars are read as one number, the first two places to the right of the separatrix as cents, the third as mills.

Thus, $32.875 is read 32 dollars, 87 cents, 5 mills.

Note.—For illustrations the *hundreds* of jackstraws already used will serve to represent dollars; the *tens*, dimes; *single sticks*, cents; and *tenths of a stick*, mills. There is plenty of room for devising other and better illustrations.

ORAL AND SLATE EXERCISES.

1. Read $1.00, $3.05, $5, $1.25, $2.75, $6.50, $4.16, $15.13, $16.12, $71.10, $35.80, $70.65, $40.50, $38.15, $49.35, $586.50, $9828.75, $16782, $0.25, $0.05.

2. Read $1.125, $3.367, $6.875, $4.126, $19.625, $18.163, $3.185, $72.05, $1.05, $1.055, $1.47, $0.876, $2.10, $342.21.

3. Write in figures, and without the aid of words, one cent, two cents, etc., to fifty cents. (Use the sign $, thus $0.01 or $.01. Arrange in columns.)

4. Write 7 dols. 6¢ 3 m., 12 dols. 70¢ 8 m., 88 dols. 60¢ 7 m., 100 dols. 9¢ 9 m., 3426 dols. 1¢ 5 m., 4870 dols. 30¢ 5 m.

Model.—7 dols. 6¢ 3 mills = $7.063.

5. Write in words, 326, $326, $326.01, 26, $0.26, $26.00, $2.60, 845, $8.45, $84.05, $0.845, $16, 160, $0.169, $16.05, $1.605. (Write abstract numbers and sums of money in separate columns.)

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6. How many cents in $1, $2, $3, $4, $5, $9, $6, $8, $7, $10, $11, $15, $17, $25, $81, $67?

7. How many cents in $1, $1.05, $1.10, $2, $2.25, $3.75, $4.85, $6.10, $4.17, $8.25, $10.20, $1.99, $16, $16.87, $35.50?

8. Write the following as cents: $1.75, $6.25, $7.35, $5.45, $7.95, $6.87, $7.21, $18.38, $19.55, $18.76, $27.87, $98.63.

Model.—$21.83 = 2183¢. Read: 21 hundred 83 cents.

9. Read the following as hundreds of cents and cents, and again as dollars and cents: 1675¢ (16 hundred 75 cents, and 16 dollars 75 cents), 500¢, 682¢, 956¢, 1871¢, 1956¢, 4563¢, 627¢, 1856¢, 3983¢, 9987¢, 8672¢, 91374¢.

10. Write the following, separating the dollars from the cents, and use the proper signs: 782¢, 396¢, 950¢, 680¢, 721¢, 354¢, 725¢, 875¢, 1982¢, 1678¢, 2585¢, 4398¢, 1875¢.

Model.—87638¢ = $876.38. (The signs ¢ and m. must be omitted when the dollar mark and the separatrix are used.)

11. How many mills in 1¢, 2¢, 3¢, 4¢, 9¢, 5¢, 8¢, 6¢, 7¢, 10¢ ? Is the mill coined ?

12. How many mills in 20¢, 70¢, 30¢, 60¢, 40¢, 90¢, 50¢, 15¢, 36¢, 58¢, 42¢, 63¢, 51¢ ?

13. How many mills in 100¢, 300¢, 600¢, 400¢, 120¢, 150¢, $1, $2, $3, $16, $9, $1.20, $1.50, 45¢, 450¢, $4.50, $9, $10, $15, $19, $18.25, $25.62, $342.15, $2100.50 ?

14. *Express as mills :*

17¢	163¢	$1.75	$15	$15.12	$185.16	$232.132
20¢	245¢	$2.25	$35	$23.19	$198.21	$830.012
25¢	350¢	$3.50	$78	$62.20	$321.15	$700.019
83¢	425¢	$4.05	$96	$70.00	$568.00	$109.346

Models.—16¢ = 160 m. ; $1.35 = 1350 m. ; $275.18 = 275180 m.

15. Read the first three columns below as cents, or as cents and mills ; the last three as dollars, cents, and mills :

10 m.	15 m.	152 m.	2645 m.	1325 m.	20175 m.
20 m.	26 m.	235 m.	1875 m.	2515 m.	38625 m.
40 m.	55 m.	428 m.	1625 m.	7628 m.	55825 m.
50 m.	32 m.	632 m.	4935 m.	4932 m.	70362 m.

16. Write the sums expressed above, using the dollar sign and the separatrix.

Model.—10 m. = $0.01. 70362 m. = $70.362.

91. By this time the pupil will perceive that any sum of United States money may be read in different ways. For instance, $1 may be read as 100¢ or as 1000 m.—$16.85 may be read as written, or as 1685¢, or 16850 m.

92. The whole process of changing one denomination in United States money into another consists in annexing 0's, or in placing or removing the separatrix. If any denomination lower than dollars is to be expressed, cents, including dimes, must always have *two places*, even though they be filled with 0's. If the number of cents be less than 10, the figure next to the separatrix must be a 0. It is very important to remember this.

Addition and Subtraction of United States Money.

93. The following rule needs no introduction:

Rule.—**Write the numbers to be added or subtracted so that the separating points shall be in a vertical line. Then proceed as in addition or subtraction of simple numbers.**

SLATE EXERCISES.

1.	**2.**	**3.**	**4.**	**5.**
$ 3.75	$ 7.30	$ 3.25	$ 51.32	$661.50
4.52	13.15	114.29	46.39	720.70
13.30	2.99	50.01	54.91	43.35
25.09	48.75	36.36	284.03	438.97

Write in columns, and add:

6. $64.22+$1.01+$6.93+$101+$0.28+$57.05+34.01.

7. $242.32+$4.00+$91.35+$628.21+$5.05+$40+$34.10.

8.	**9.**	**10.**	**11.**	**12.**
$752.24	$324.40	$ 13.80	$13428.79	$ 12.00
6.23	276.19	14.60	12469.31	10.00
15.62	1328.45	160.32	3872.84	960.05
103.78	9327.71	369.49	92476.75	200.91
923.60	1000.01	765.30	1328.62	1342.78

Find the sum of

13. $3.421+$71.243+$62.103+$5.009+$34.24+$6.328.

14. $20.21+$342.109+$34.497+$603284.679+$9.384.

15.	**16.**	**17.**	**18.**
$ 12.345	$ 10.102	$793.104	$ 1.000
67.890	1.000	38.796	342.060
35.719	100.309	596.647	3.501
864.246	540.701	764.521	917.624
395.713	724.309	990.901	249.387
54.575	99.830	881.725	1800.000
983.600	786.250	9651.062	71.875

19. Add 17 dols. 30¢ 4 m., 123 dols. 45¢ 7 m., 129 dols. 37¢ 9 m., 98 dols. 11¢ 6 m., 59 dols. 24¢ 7 m., 3482 dols. 99¢ 9 m.

20. Find the sum of \$5.097 + \$0.083 + \$5.629 + \$15.912 + \$4.691 + \$59.03 + 16.001 + \$18.875.

21. Find the sum of \$0.187 + \$9.987 + \$4.46 + \$0.365 + \$73.28 + \$83.18 + \$1. + \$5000 + \$3.057 + \$15.01.

22. Add five hundred sixty-nine dols. forty-seven cents, seven thousand two hundred ninety-eight dols. sixty-three cents five mills, ninety-six dols. fifty-three cents seven mills, one hundred dols. forty-five cents seven mills, nine hundred dols. ten cents, one dol. fifty cents.

Applications.—23. A gentleman owes the following: To Mr. Brown, \$16.50; to Mr. Jones, \$79.54; to Mr. Thomas, \$82.35; to Mr. Alfreds, \$46.83. What is the sum of these debts?

24. Mr. Willey gave the following sums to be used for charitable purposes: \$3470.25; \$6945.75; \$1015.00; \$5900.25. How much did he give in all?

25. The property of Mr. Childs was valued as follows: Land, \$3560; house, \$1834; barn, \$690; horse, \$180; carriage, \$135; tools, \$382.95; money in bank, \$987. How much was he worth?

26. Mr. Johnson bought a set of furniture for the parlor, costing \$385.75; one for the sitting-room, costing \$229.55; one for a bed-room, costing \$176.25; one for the dining-room, costing \$194.40. What was the total cost of the four sets?

Subtract:

27.	**28.**	**29.**	**30.**	**31.**
\$642.67	\$648.16	\$3247.67	\$162.38	\$6235.41
83.90	59.23	843.98	37.84	1987.93

32.	**33.**	**34.**	**35.**	**36.**
\$639.10	\$794.13	\$9.00	\$85.00	\$9500.05
234.78	197.25	2.97	3.78	3428.69

37.	**38.**	**39.**	**40.**	**41.**	**42.**
\$953.24	\$123.45	\$523.42	\$6.932	\$173.00	\$11.000
735.38	67.89	94.51	0.478	56.43	9.345

43–50. Deduct $87.93 from each of the following: $100.00, $93.13, $703.24, $103.10, $1834.29, $793.11, $600, $87.94.

51–55. Deduct $934.782 from the footings of examples 8–12.

56–59. Deduct $379.999 from the footings of examples 15–18.

Subtract:

60.	61.	62.	63.
$762.949	$564.120	$24.500	$724.380
438.758	259.437	19.999	149.871

64. Take four thousand six hundred forty-five dollars twenty-eight cents three mills from six thousand two hundred thirty-eight dollars eleven cents.

Applications.—65. If a person's property amounts to $7434.90, and his debts to $1350.78, how much is he worth?

66. Mr. White opened his store with goods worth $6100.50. Six weeks afterward he took an inventory (list) of goods remaining on hand, and found their value to be $4417.46. How much had he disposed of?

67. One season Mr. Hardy, who made a business of dealing in real estate (houses and lands), bought:

a. A city house and lot for $8765, and sold them for $10000;

b. A farm for $16850, and sold it for $17050;

c. A business block for $78500, and sold it for $82000;

d. Two city lots for $30500, and sold one for $13250 and the other for $18525;

e. A city lot for $75000 and sold it for $83500;

f. A theatre for $125250, and sold it for $121750. Find what he gained or lost on each transaction. Did he gain or lose by the business of the season, and how much?

---

68. How many cents are there in one half dollar? In one quarter of a dollar? In one fifth of a dollar? In one tenth of a dollar? In three quarters? In four quarters? In two quarters? *In two fifths?* In three tenths?

69. Write the following sums as dollars and cents: $\$1\frac{1}{4}$, $\$2\frac{1}{2}$, $\$3\frac{1}{5}$, $\$5\frac{3}{4}$, $\$18\frac{1}{2}$, $\$22\frac{2}{5}$, $\$28\frac{3}{5}$, $\$32\frac{3}{4}$, $\$42\frac{2}{4}$, $\$48\frac{4}{5}$.

Model.—$\$1\frac{1}{4}=\$1.25$.

70. Write the number of mills equivalent to $12\frac{1}{2}$¢, $18\frac{2}{5}$¢, $37\frac{1}{2}$¢, 75¢, $43\frac{3}{5}$¢.

Model.—$\frac{1}{2}$¢=5 m., $12\frac{1}{2}$¢=12¢5 m. = 125 m.

71. Find the sum of $\$1\frac{3}{4}$, $\$3\frac{1}{2}$, $\$5\frac{3}{4}$, $\$7\frac{2}{4}$, $\$90\frac{1}{2}$, $\$10\frac{1}{2}$. (Write \$1.75 for $\$1\frac{3}{4}$, etc.)

72. Add $\$7\frac{2}{5}$, $\$8\frac{1}{2}$, $\$1\frac{5}{10}$, $\$3\frac{7}{10}$, $\$6\frac{1}{2}$, $\$30\frac{3}{5}$.

Applications.—**73.** William has $\$7\frac{1}{2}$ in his savings bank, Bertha $\$2\frac{3}{4}$, Carl $\$1\frac{1}{2}$, May $\$3\frac{1}{2}$, Emma \$0.50. How much have they in all?

74. Mrs. Duncan paid the butcher on Monday \$0.60; on Tuesday, \$0.78; on Wednesday, $\frac{1}{2}$ dol.; on Thursday, $\frac{3}{4}$ dol.; on Friday, $\frac{2}{5}$ dol.; and on Saturday, \$1.10. How much in all?

75. My new Reader cost $\frac{4}{5}$ dol., my Speller $\frac{2}{5}$ dol., my Arithmetic $\frac{1}{2}$ dol., and my new slate 15¢. How much did I pay for them all?

76. I bought a pair of shoes for $\$4\frac{1}{2}$, a hat for $\$7\frac{1}{2}$, and an umbrella for $\$1\frac{3}{4}$, and gave the merchant a \$20 bill. What change did I get?

77. In paying an account of $\$34\frac{4}{5}$, I gave the dealer 4 ten-dollar bills. What change was due me? If I had given him a \$50 bill, what change should I have received?

78. A boy had bought four articles at the grocery worth \$2.25. But not having so much money, he handed back one of the articles costing $\frac{9}{10}$ of a dollar. How much did he pay for the other three articles?

79. John buys coffee for $\$1\frac{1}{5}$, sugar for 70¢, cheese for $\frac{1}{2}$ dollar. He sells the grocer potatoes for \$1.50. How much does John have to pay, after deducting the price of the potatoes from his bill?

Multiplication of United States Money.

Illustrative Example.—1. If the price of rye is $1.355 per bu., what will 65 bushels cost?

Suggestive Questions.—If $1.355 were written in column 65 times, and an addition of all made, where would the separatrix fall? How many places would there be to the right of the point? What denominations would they represent?

Or, How many mills are there in $1.355? In 65 times $1.355? Reduce the result to dollars. How many places to the right of the point? What denominations do they represent?

Operation.

$1.355 price per bu.
65 no. of bushels.
6775
8130
$88.075 cost of 65 bu.

94. ***Rule.***—**Multiply as in simple multiplication. The product will be of the same denomination as the lowest order of the multiplicand. If it be in cents or mills reduce to dollars and prefix the dollar mark.**

SLATE EXERCISES.

2–7. Multiply $87.74 by 27.—$324.034 by 56.—$1.95 by 18.—$27.341 by 35.—$0.934 by 746.—$0.34 by 61.

8–22. Multiply each of the following sums of money by 8; by 37; by 368: $0.398; $20.03; $47.731; $621.70; $0.604.

23–37. Find first 13, then 49, then 387 times $34.75; $967.03; $309.08; $7654.60; $190.10; and subtract $99.999 from each product.

38. What is the product of $4.37 by 18×27×15?

Applications.—**39.** How much must the government pay for 327 horses, at $135.50 each?

40, 41. What will 426 sheep cost at \$4.87$^1/_2$ per head? At \$3.62$^1/_2$? (Express fractions of cents in mills.)

42. What is the amount of a contribution if 157 persons contribute each \$5$^1/_4$?

43–45. What will 476 lb. of tea cost at 65¢ a lb.? At 50¢? At $^3/_4$ dol.?

46. A merchant bought 3 bales of cloth containing respectively 485, 492, and 497 yards, at \$1.87 $^1/_2$ a yard. Find the cost ?

47. The merchant just spoken of sold the two bales first mentioned at \$2.25, and the last one at \$2.75 per yard. What did he receive for the cloth ?

48. A grocer buys 38 doz. loaves of bread on Monday, 35 doz. on Tuesday, and 36 doz. on each remaining day of the week, at 60¢ per doz., and sells at 7¢ per loaf. How much does he gain ?

49. A grocer bought 9 barrels of cider, each barrel containing 30 gallons, at 12 $^1/_2$¢ per gallon. What did the cider cost ?

50. What is the cost of 4 barrels of sugar, weighing 495 lb. each at 7 $^1/_2$¢ a pound ?

51. What is the cost of 156 acres of land at \$20.50 per acre ?

52. Mr. Jones bought of Messrs. Taylor, Kilpatrick and Co., 16 yd. silk, at \$2.25 a yd.; 6 yd. gingham, at \$0.19 a yd.; 27 yd. linen, at \$0.37 a yd.; 39 yd. muslin, at \$0.13 a yd. What was the amount of the bill ?

53. Mr. Taylor bought of Mr. Watts the following implements : a spade for $^3/_4$ dol., a rake for $^3/_5$ dol., a hoe for $^1/_2$ dol., a shovel for 45¢, and a lawn-mower for 14$^3/_4$ dols. Find the amount.

54. Mr. James went to market with \$5. He spent for eggs 45¢, for butter 98¢, for fruit 25¢, and for flour \$1.20. How much had he left of the \$5 ?

55. A laborer earns \$50 and spends \$39.75 a month. How much will he have saved at the end of six months ?

56. What are the profits of a concert, if 3427 tickets are sold at \$1 $^1/_2$ each, and the expenses are \$938.40 ?

57. Mr. Mills sold 837 shade-trees for \$0.65 each. How much did he receive for the lot ?

58. A drover bought 265 head of cattle at \$43.75 a head, and paid \$8.35 a head to get them to market, where he sold them at \$56.80 a head. How much did he gain by the transaction ?

Division of United States Money.

ILLUSTRATIVE EXAMPLES.

1. If a builder pays $693.68 for lumber at $2\,{}^1/_2$¢ per foot, how many feet does he buy?

```
        27747 5/25 times.
  25 m.)693680 m.
        50
        193
        175
         186
         175
          118
          100
           180
           175
             5
```

Note 1.—Every time $2\,{}^1/_2$¢ can be taken from $693.60, a foot of lumber can be bought. Hence, to find the number of feet, we find how many times $2\,{}^1/_2$¢ is contained in $693.68. To do this, we change both the $2\,{}^1/_2$¢ and the $683.68 to mills, and divide the second by the first.

A shorter way would be to change both sums to half cents, and divide, but the first way is generally the better. Try both and see which you like best.

2. If $693.68 be equally divided among 25 men, what will be the exact share of each?

```
       $27.747 5/25
  25)$693.68
      50
      193
      175
       186
       175
        118
        100
         180
         175
           5
```

Note 2.—If $693 be divided into 25 equal parts, there will be $27 in each part, and $18 undivided. $18 = 180 dimes, 180 dimes + 6 dimes = 186 dimes. If 186 dimes be divided into 25 equal parts, there will be 7 dimes in each, and 11 dimes will remain undivided. To this we add the 8¢, and then proceed as before till we come to a remainder of 5 mills. For the present, remainders should be disposed of as directed in Art. 76, p. 80.

Note 3.—Thus we find that division of United States money is, first, the process of finding how many times one sum of money is contained in another; and, second, of finding a required part of a given sum. (See also Art. 75, p. 78.)

95. ***Rule I*** **—To find how many times one sum is contained in another, change both to the lowest denomination in either, and divide as in simple division. The quotient will be in integers.**

Rule II **—To find a required part of a given sum of money, divide the sum by the number of parts, as in simple division. The place of the separatrix in the quotient will be directly over the *separatrix* of the dividend.**

SLATE EXERCISES.

3. Divide $100.50 by 5. Also by 7, by 9, by 65.

4, 5. Four persons are to have equal shares of $4412.88. How much will each one receive? How much would each receive if there were 12 persons?

6–11. Divide $369.009 by 3, by 9.—Also by 8, by 6, by 5, by 15.

12–16. Divide $3759.91 by 19, by 35, by 54, by 67.—Are you here required to find certain parts of $3759.91; or, how many times that sum contains the several divisors?

17. Which is greater, $^1/_5$ or $^1/_2$ of $90.50? How much?

Find

18. $^1/_2$ of $ 97.78.
19. $^1/_4$ of $ 363.68.
20. $^1/_5$ of $ 728.15.
21. $^3/_5$ of $8257.25.
22. $^1/_{16}$ of $8775.36.
23. $^7/_9$ of $ 436.50.
24. $^5/_7$ of $ 410.41.
25. $^3/_8$ of $9873.56.
26. $^{13}/_{16}$ of $10391.52.
27. $^{15}/_{27}$ of $ 8335.71.
28. $^{11}/_{12}$ of $ 5654.76.
29. $^{18}/_{25}$ of $ 9864.75.

Applications.—30–31. How much sugar can be bought for 99¢ at 11¢ a lb.? At 9¢ a lb.?

32–35. If oranges cost $4^1/_2$¢ apiece, how many can be had for $3.60, for $3.78, for $36.00, for $0.90?

36. A dozen chairs can be bought for $11.40. How much does 1 chair cost?

37. A carpenter has 34 men at work; at the end of the week, 17 of them receive $211.65 wages; the other 17 receive only $184.45. How much does each one of the two classes of workmen receive per week?

38. A street commissioner had 347 men at work. At uniform wages the weekly pay roll amounted to $2602.50. What did each man receive per day?

Six workmen received pay for 26 days' work as follows:

39. The carpenter, $55.25.
40. Painter, $52.
41. Bricklayer, $65.
42. Plumber, $68.25.
43. Laborer, $42.25.
44. Plasterer, $63.70.

What were the daily wages of each?

Miscellaneous Examples.

1. Mr. Jacobs paid me \$27.34; Mr. Niel, \$79.14; Mr. French, \$34.27; Mr. Myers, \$647.79. My expenses on the tour of collection were \$19.68. When I started out I had \$50.75 in my purse. How much money ought I to have had on my return.

2. In 1873 I paid \$52.23 taxes; in 1874, \$50.79; in 1875, \$46.27; in 1876, \$44.83; in 1877, \$42.21; and in 1878, \$40.90. How much in these 6 years? The repairs on my house in the meantime cost \$238.65. For the first 3 years I received \$650 per year rent, for the last 3 years \$700 per year. How much did I receive in the 6 years clear of expenses?

3. A lady had \$30. She bought a dress for \$9.15, shoes for \$3.40, a bonnet for \$4.50, and 23 yd. muslin at 25¢ a yd. How much did she have left?

4. A farmer owed \$500, and gave in part payment 435 bu. wheat, at \$1.02 a bu. How much money was yet due?

5. If you spend \$0.74 a day, how much will you save in a year if your salary is \$475? (365 days to the year.)

6. If a laborer works 60 days, 10 hours a day, and receives \$135 for his labor, how much does he earn per hour?

7. Mr. Jordan sells Mr. Marsh the following articles: 3 cashmere long shawls, at \$45.75 each; 45 yd. black satin, at \$2.20 a yd.; 12 alpaca umbrellas, at \$1.35 apiece; 60 worsted cord and tassels, at \$0.35 apiece. Find the cost.

Note.—In the following memoranda the price per pair, yard, doz., or single article is given as usually written. The pupil is required to extend the items, and find the footings. The sign @ stands for the word *at* (that is, per pair, per lb., etc.).

8. *Find the cost of*

3 pr. kid gloves,	@	\$1.35
17 yd. Malta lace,	@	.76
4 doz. handkerchiefs,	@	1.80
1 doz. pair linen cuffs,		1.56
6 yd. silk fringe,	@	1.10
8 ostrich plumes,	@	3.25
8 pr. linen gloves,	@	.65

9. *How much must be paid for*

5 gal. vinegar,	@	\$.27
15 lb. cheese,	@	.09
3 lb. hominy,	@	.45
27 lb. sugar,	@	.07
22 lb. soap,	@	.06
3 gal. molasses,	@	.70
6 lb. prunes,	@	.16

10. If you are sent with a $2 bill to the bakery to get 1 doz. rolls, at 1¢ apiece; 3 loaves of bread, at 8¢ a loaf; 3 doz. cookies, at 10¢ a doz.; and 10¢ worth of caramels, what change will you bring back?

11. Mother makes the following purchase at the crockery store: 1 cream pitcher, 75¢; 1 sugar bowl, 45¢; $^1/_2$ doz. plates, at $1.20 a doz.; $^1/_2$ doz. egg cups, at 90¢ a doz.; 3 cups and saucers, at 60¢ a pair; 2 doz. fruit jars, at 9¢ apiece. What is the bill?

12. If your mother sends you to the grocery with $5 to buy $^1/_2$ lb. of tea, at 90¢ a lb.; 1 lb. of coffee, at 40¢ a lb.; 5 lb. of granulated sugar, at 11¢ a lb.; 3 lb. of lump sugar, at 12¢ a lb.; 1 small bag of salt, at 9¢; 4 loaves of bread, at 6¢ a loaf; 1 peck of apples, at 80¢ a bu., what change will you receive?

13. If you are sent with $2 to buy 3 lb. rice, at 10¢ a lb.; 20 lb. of flour, at 5¢ a lb.; 6 lb. of cheese, at 9¢ a lb.; 5 lb. of prunes, at 16¢ a lb.; 1 gal. coal-oil, at 8¢ a quart, will you have money enough? If not, which article must you omit to keep the sum total within $2?

14. Twelve tons of coal cost $75.00, how much is that per ton? What would 37 tons cost at that rate?

15. A person sells 5 cows at $55 each, and a yoke of oxen at $125. He agrees to take in payment 80 sheep. How much do the sheep cost him per head?

16. What is the cost of 39000 feet of planed pine lumber at $40 per thousand feet? Of 16000 shingles at $2.75 per thousand?

17. *Find the cost of*

60 pr. overshoes,	@	$ .65
15 " boots,	@	4.25
17 " gaiters,	@	3.85
37 " slippers,	@	1.75
230 " mittens,	@	.34
2 doz. pr. slippers,	@	9.00
2 pr. boots,	@	9.50

18. *Required the cost of*

9 lb. lard,	@	$.08
15 lb. butter,	@	.22
18 lb. pork,	@	.06
20 lb. rice,	@	.09
12 lb. raisins,	@	.20
4 cans oysters,	@	.35
10 lb. codfish,	@	.10

19. Mr. White bought 6 tubs of butter, containing 58 lb. each, for $80.04. How much did he pay per lb.?

20. He sold the butter at a profit of 12¢ a pound. Deducting 7 lb., which he used in his family, how much did he get for it?

21. When coal is $\$4^1/_4$ per ton, how many tons can be bought for $238? How much would be saved by buying at $4 per ton?

22. A farmer buys goods amounting to $235.75. He pays in cash $58.25, and agrees to pay the balance in rye, at $1.25 a bushel. How many bushels will be required?

23. How many pounds of cheese, at 15¢ a lb., must be given in exchange for 14 yd. of gingham, at 30¢ a yard?

24. Subtract $37.87 from $237.37; from the remainder subtract $37.87, and continue subtracting till the remainder is less than the subtrahend. What is the remainder? Is this the shortest way to find the remainder?

25. Multiply $4.35 by 2; multiply the product by 3; multiply the second product by 4; the next by 5; the next by 6; and the next by 7. What is the last product?

Arrange each line in column, and add:

26. $13.44, $300, $55.25, $288.39, $19.50, $31.67, $509.07.

27. $67.31, $180.61, $79.03, $152.70, $14.23, $11.12, $50.22.

28. $88.75, $264.16, $44.56, $76.82, $30.50, $72.39, $142.33.

29. $10.13, $7.56, $2.18, $55.44, $11.19, $70.25, $312, $9.

30. $13.33, $72.69, $15.437, $34.805, $125.595, $77.666.

31. Add together all the sums of money given in examples 26 to 30, inclusive.

32. A store-keeper, who was about to pay some debts, found that he had $37.45 in change and $76 in bank-notes in his money-drawer, $318 in his safe, and $98.36 in his pocket-book. How much had he left after paying 5 bills of $56.10, $38.05, $48.00, *$213, and $78.90,* respectively?

33. The treasurer of a street railroad took 400 dimes, 800 quarter-dollars, 23 twenty-cent pieces, 600 half-dollars, 1000 five-cent pieces to be exchanged for $5 bills. How many did he get?

34. A grocer exchanged bills for small change. How many 5¢ pieces could he get for $5, $10?—How many dimes for $5, $10?—How many quarters for $25, $45?—How many half-dollars for $37, $54, $96?

Making Change.

96. 1. You buy three pounds of rice at 9¢ a pound, and hand the grocer in payment a dollar bill; how does he count the change due you?

Answer.—Giving you the rice he would count that as 27¢; then placing in your hand successively 3¢, 10¢, 10¢, and 50¢, he would count *30, 40, 60 $1.* This is the most convenient way, and least liable to error. It is similar to the "making up" method in subtraction, which is recommended on page 41.

2. Having only 5¢ and 10¢ pieces, how will the change be counted, taking 15¢ out of $1?

3. Having 1¢, 5¢, 10¢ pieces, and $1 bills, how would you make the change for 35¢ out of $2? For 85¢ out of $5? For 75¢ out of $10? For $1.12 out of $1.50? For $6.03 out of four two-dollar bills? For 67¢ out of $2? For $3.33 out of $5?

4. If the merchant has no change except 25¢, 50¢, and $1 pieces, how can he make change for $2.75 out of $5?

5. A collector presents a bill for $1.90; you have only two $1 bills, one 10¢, and one 5¢ piece = $2.15. The collector has only large bills and quarter dollars. How can the change be made? (If you were to give him your $2.15, could he then make the change?)

6. You owe $2.75, but have only three dollars and a quarter. How can change be made, the collector having none less than a half dollar?

7. If a grocer has only small change, namely, 1¢, 2¢, 3¢, 5¢, 10¢, 20¢, 25¢, 50¢ pieces, how can he make change for $2, the goods you have bought costing 16¢?

8. If a merchant has only \$1, \$2, and \$5 bills, and 1¢ and 5¢ pieces, how will the change for \$3.27 be counted out of a \$20 note?

9. Having 1¢, 5¢, 25¢, and 50¢ pieces, and \$2 bills, how can you count the change for \$1.15 out of \$5? For \$2.23? For \$3.45? For \$1.84? For \$4.66? For \$1.11? For 92¢? For \$4.46?

10. Having only 1¢, 10¢, 25¢, and \$1 pieces, how can you count the change for 27¢ out of \$2? For \$1.34 out of \$5?

Count out the proper change in each of the following transactions:

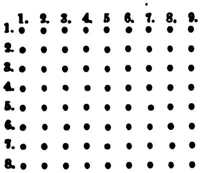
	Goods sold.	Money received.
11.	5 lb. coffee, @ 32¢; 5 lb. sugar, @ 9¢; 3 lb. cheese, @ 14¢.	\$5
12.	2 lb. beef, @ 19¢; 2 lb. butter, @ 42¢; radishes, 10¢.	2
13.	3 lb. soap, @ 10¢; 2 lb. starch, @ 12¢; 1 paper allspice, 12¢.	1
14.	$^1/_2$ lb. tea, @ 90; 4 lb. sugar, @ 11¢; 1 qt. strawberries, 25¢.	2
15.	1 lead-pencil, 10¢; envelopes, 10¢; 1 daily newspaper, 3¢.	5
16.	5 yd. muslin, @ 13¢; 3 spools cotton, @ 6¢; 6 handkerchiefs, @ 28¢.	4
17.	900 lb. pork, @ 3¢; 5 bu. peaches, @ \$2.50; 7 bu. apples, @ 75¢.	50
18.	18 bu. oats, @ 50¢; 12 bu. corn, @ 75¢; 13 cwt. hay, @ \$1.05.	35
19.	3 pk. peaches, @ \$2 per bu.; 5 qt. cherries, @ 18¢; $1^1/_2$ lb. butter, @ 48¢.	5

CHAPTER VIII.

FACTORS AND DIVISORS.

Definition.

97. An ***Integer*** is a whole number. It is so called to distinguish it from a fraction. (This chapter treats of integers only.)



Factors.

Having counted by 8's and 9's to 72, the pupil learned that

8 times 9=72, and
9 " 8=72.

Then he learned—

1. That 8 and 9 are called factors of 72.
2. That 72 is called the product of 8 and 9; and
3. That 72 is called a multiple of 9; also of 8.

98. But since 8 times 9=72 and 9 times 8=72, nine is contained exactly 8 times, and 8 exactly nine times in 72. Thus the factors of a number are exact divisors of that number.

99. Hence, to find the factors of a given number we ascertain by trial what numbers will divide it without a remainder; the divisors and quotients are the factors sought.

Thus we obtain 7 different pairs of factors of 210, as follows:

2)210	3)210	5)210	6)210	7)210	10)210	14)210
105	70	42	35	30	21	15

Definitions.

100. A ***Factor*** of a number is any one of two or more integers which, multiplied together, produce the number.

101. When one number can be divided by another without remainder, the dividend is said to be ***divisible*** by the divisor, and the divisor is called a ***Measure*** or ***Exact Divisor*** of the dividend.

102. A number that is the product of other factors besides *itself* and *one* is called a ***Composite Number.***

Note 1.—A *Composite* number is so called because it is *composed* of other factors.

Note 2.—Since a composite number is divisible by its factors, it may be defined to be a number that is divisible by other numbers besides itself and one.

103. A ***Prime Number*** is one that has no factors and hence no exact divisor except itself and 1.

104. A ***Prime Factor*** is a factor which is a prime number.

105. An ***Even Number*** is one that is divisible by 2.

106. An ***Odd Number*** is one that is not divisible by 2.

SLATE EXERCISES.

1. Write in columns the numbers in order from 1 to 35; also from 36 to 70, inclusive, and opposite to each write all the pairs of factors that will produce it. Thus

$1=1\times 1$
$2=1\times 2$
$3=1\times 3$
$4=2\times 2$
$5=1\times 5$
$6=2\times 3$
etc.

$36=2\times 18,\ 3\times 12,\ 4\times 9,\ 6\times 6$
$37=1\times 37$
$38=2\times 19$
$39=3\times 13$
$40=2\times 20,\ 4\times 10,\ 5\times 8$
$41=1\times 41$
etc.

2. In the same manner write the pairs of factors of numbers from 71 to 107, inclusive; also from 108 to 144.

3. Make a table such as the one required in Ex. 5, p. 53, *omitting the first* line and first column. Give the factors orally.

4. Make a separate list of numbers from 1 to 144 that have

2 for one or more of their factors. *Notice* that the right-hand figure of each is — — — — or —. (?)

5 for one or more of their factors. *Notice* that the right-hand figure of each is — or —. (?)

3 for one or more of their factors. Divide the sum of the digits of each of these numbers by 3, and notice the remainder, if any.

9 for one or more of their factors. Divide the sum of the digits of each by 9, and notice the remainder, if any.

Thus we discover some

AIDS IN FINDING FACTORS.

107. It may be shown to be true of any number that it has

2 for a factor if the right-hand figure is 2, 4, 6, 8, or 0;

5 for a factor if the right-hand figure is 0 or 5;

3 for a factor if the sum of its digits is divisible by 3;

9 for a factor if the sum of its digits is divisible by 9.

Apply the foregoing aids in the following exercises:

1. Tell which of the dividends at the top of p. 82 are divisible by 2; by 5; by 3; by 9.

2. Write 10 numbers of three or more figures each, all of which shall be divisible by 2; by 5; by 3; by 9.

3. Change one figure in each of the following numbers, so as to make the number divisible by 2: (State what change you make, and why.)

4379	6479	5243	7957	4343
5627	8123	2147	8971	5557
8291	4871	9281	3629	4441

4.–6. Change one figure in each, so that the number shall be divisible by 5.—Change the last figure in each, so that the number shall be divisible by 3.—Change the first figure in each, so that the number shall be divisible by 9.

Factoring.

108. Since 7 is a factor of 14, it must be a factor of any number of times 14, as 28, 42, etc. Thus it is true always that

A factor of a factor-of-a-number is a factor of the number itself.

Hence, having obtained one prime factor of a number directly from the number itself, a second one may be obtained from the quotient of the first, and a third, if any, from the quotient of the second, etc. Thus

Solution.

$$\begin{array}{r|l} 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ & \ \ 7 \end{array}$$

Proof.

$2 \times 3 \times 5 \times 7 = 210$

Explanation.—2 being contained 105 times in 210, 2 and 105 are factors of 210. Then dividing the quotient by 3, we find that 3 and 35 are factors of 105, and hence also of 210; and, again, finding that 5 and 7 are factors of 35, we know that they are factors of 105 and also of 210. Thus we derive the

109. ***Rule.*—Divide the given number by any prime factor, and if the quotient is not a prime number, divide it in like manner, and so continue to divide till the quotient is a prime number. The divisors and the last quotient are the factors sought.**

SLATE EXERCISES.

Find the prime factors of

1. 1050
2. 2625
3. 1820
4. 1485
5. 1155
6. 5985
7. 4620
8. 4802
9. 5432
10. 7000
11. 8140
12. 8712
13. 1320
14. 1768
15. 1848
16. 1906
17. 1858
18. 1478
19. 2956
20. 2406
21. 2526
22. 2978
23. 2992
24. 3936
25. 3430

Note.—The learner will avoid useless labor if he will keep it in mind that the quotient is as much a factor of the dividend as the divisor itself.

Suppose, for instance, that he is working to find the prime factors of 479, as in the last of the preceding examples. He tries successively every prime number from 2 upward, till he comes to 23, when he finds that the quotient has become less than the divisor. Here, if he stops to think, he will say to himself: "It is of no use to try any further. This number can not have an exact divisor greater than 23, for if it had, it would have another less than 23; but I have tried every prime number from 2 to 23, and I know it has none. This number is prime."

Common Factors and Common Divisors.

110. A factor that occurs in each one of two or more numbers is a ***Common Factor*** of those numbers. Thus 15=3×5 and 21=3×7; the factor 3 is common to 15 and 21.

Note.—A factor is said to be *common* to two or more numbers, just as we may say that the letter *i* is common to all the syllables of the word *Mississippi.*

111. *A factor of a number being an exact divisor of the number, a factor that is common to two or more numbers is a common divisor of those numbers.*

Find the prime factors of 252 and 2310, and show that the common factors are common divisors of those numbers.

112. *The product of any two or more prime factors of a number being an exact divisor of that number, the products of the prime factors common to two or more numbers are common divisors of those numbers.*

Show that the products of the prime factors that are common to 294 and 315 are common divisors of those numbers.

113. *Since an exact divisor of a number can have no factor which does not occur in that number, the product of all the prime factors that are common to two or more numbers is* ***the greatest common divisor*** *of those numbers.*

Find, if you can, any other divisors of 396 besides its prime factors, and the products of two or more of them. Can 1820 and 2310 have any other common divisors than their common factors and their products? Try to find one.

114. Hence, to find the greatest common divisor of two or more numbers—

Rule.**—1. Resolve the given numbers into their prime factors.**

2. Multiply together all the factors that are common to all the numbers. The product will be the greatest common divisor sought.

Example.—Find the g. c. d. of 546 and 910?

Prime Factors Found.

2	546
3	273
7	91
	13

2	910
5	455
7	91
	13

Prime Factors Arranged.

546=2×3×7×13

910=2×5×7×13

Common Prime Factors Multiplied.

2×7×13=182 g. c. d.

115. A shorter Process.—When the prime factors are readily detected, the process is somewhat simplified and considerably shortened by dividing the given numbers only by the factors that are common, and multiplying these together for the greatest common divisor. The principle of this operation is the same as that of the one given at the bottom of the preceding page.

Operation.

2	924	990
3	462	495
11	154	165
	14	15

$2 \times 3 \times 11 = \mathit{66}$ g. c. d.

Note.—The product of any two of the common prime factors is a common divisor, but it requires the continued product of ALL of them to make the *greatest common divisor.*

ORAL EXERCISES.

Find the greatest common divisor of

1. 14 and 21
2. 26 and 39
3. 40 and 56
4. 72 and 99
5. 80 and 48
6. 57 and 69
7. 15 and 93
8. 50 and 75
9. 56 and 84
10. 21 and 56
11. 32 and 64
12. 34 and 38
13. 58 and 87
14. 18 and 82
15. 81 and 45
16. 46 and 28
17. 36 and 54
18. 49 and 98
19. 54 and 81
20. 63 and 81

SLATE EXERCISES.

Find the greatest common divisor of

21. 323 and 425
22. 228 and 399
23. 615 and 735
24. 819 and 945
25. 949 and 871
26. 7008 and 7968
27. 7568 and 3784
28. 3876 and 1983
29. 7956 and 7668
30. 7378 and 9758
31. 7992 and 9900
32. 8100 and 6300
33. 9864 and 9528
34. 6144 and 6930
35. 4374 and 5508

Find the greatest common divisor of

36. 45, 57, and 81
37. 63, 99, and 126
38. 36, 54, and 56
39. 72, 84, and 90
40. 306, 408, and 510
41. 420, 462, and 84

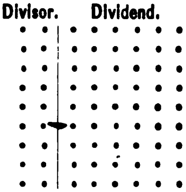
Find the largest number that will exactly divide

42. 546, 462, and 882
43. 900, 936, and 2520
44. 19635, 5355, 8925, and 12495
45. 19782, 16485, and 14287

Cancellation.

116. The principle that dividing divisor and dividend by the same number does not alter the quotient may be demonstrated as follows:

Explanation.—Any divisor and dividend having a common factor may be represented by an equal number of *rows* of dots, as 18 and 54 at the right. Whence it becomes evident that *any part* of the divisor as one or more lines is contained in a *like part* of the dividend, as many times as the *whole* divisor is contained in the *whole* dividend.



117. Hence any factor or number of factors common to divisor and dividend may be rejected without affecting the quotient.

That this principle may be well impressed upon the mind, let many examples, such as the following, be worked out:

Example.—1. (*a*) Divide the product of 7, 3, 13, 5, 7, 19, and 3, by the product of 7, 3, 7, 5, and 3. (*b*) Reject all common factors, find the product of the remaining ones and divide.

Note.—The factor 1 always remains in place of factors omitted. It is not written because it does not affect the result.

Example.—2. How many bushels of oats at 48¢ a bu. can Mr. A. get for 3 crocks of butter containing 8 lb. each at 32¢ a lb.

Solution.

$$\begin{array}{r} 32\text{¢} \\ 8 \\ \hline 256\text{¢} \\ 3 \\ \hline \$7.68 \end{array} \qquad \begin{array}{r} 16 \\ \hline 48\text{¢})768\text{¢} \\ 48 \\ \hline 288 \\ 288 \end{array}$$

Analysis.—At 32¢ a pound, 8 lb. of butter will cost 8 times 32¢ = 265¢, and 3 crocks containing 8 lb. each will cost 3 times 256¢ = $7.68, and as many bushels can be bought for $7.68 as there are times 48¢ in $7.68 = 16.

But we may indicate this work by writing the factors (makers) of the dividend on the right and the divisor on the left side of a vertical line, and shorten the work by rejecting common factors, as shown at the right.

$$\begin{array}{r|l} \cancel{3}\ \cancel{48} & \cancel{32}\ 2 \\ & \cancel{3} \\ & 8 \\ \cline{2-2} & 16\ \textit{Ans.} \end{array}$$

118. To ***cancel*** is to erase or cross out, hence the word ***Cancellation*** is applied to erasing or crossing out factors and terms which counterbalance each other in an arithmetical operation.

SLATE EXERCISES.

1. $$\frac{15\times 57\times 36\times 35}{12\times 19\times 25}=$$

2. $$\frac{17\times 95\times 8\times 23}{85\times 38\times 5}=$$

3. $$\frac{42\times 18\times 65\times 11}{35\times 45\times 13}=$$

4. $$\frac{91\times 36\times 94\times 54}{78\times 14\times 18}=$$

5. How many cheeses, weighing 49 lb. each, at 12¢ a pound, must be given in exchange for 13 barrels of flour at 4¢ a pound? (196 lb. = 1 barrel of flour.)

6. How many sacks of wheat, containing 3 bu. each, at 96¢ a bushel, must be given for 78 sacks of potatoes, each containing 2 bu. at 64¢ a bu.?

7. A lady bought 9 yards of ribbon at 56¢ per yard, but exchanged it for other ribbon at 32¢ per yard; how many yards did she then get?

8. At $129 for 27 acres of land, what will 180 acres cost?

9. At what price per yard will 5 bales of cloth, containing 12 pieces of 42 yards each, pay for 50 rolls of carpeting, of 75 yards each, at $2.10 per yard?

10. Divide 34×102×85 by 51×17×68.

11. Divide 16773×13401×11412 by 11182×11912×8559.

How many

12.	Bu. apples @ 60¢	will pay for 35 lb. tea @ 84¢?
13.	Bu. peaches @ $3.50	“ “ “ 25 tons coal @ $12.60?
14.	Pieces muslin (39 yd.) @ 12¢	“ “ “ 26 tubs butter (72 lb.) @ 32¢?
15.	Tons hay @ $12.18	“ “ “ 3 barrels sugar (232) @ 9¢?
16.	Horses @ $91	“ “ “ 7 acres land @ $559?
17.	Cows @ $39	“ “ “ 650 sheep @ $3?
18.	Mules @ $125	“ “ “ 25 horses @ $160?
19.	Tubs butter (54 lb.) @ 28¢	“ “ “ 378 yd. muslin @ 16¢?

At what price will

20.	65 lb. coffee	pay for 26 bu. potatoes @ 45¢?
21.	75 acres land	“ “ 21 horses @ $125 each?
22.	26 barrels pork	“ “ 78 bl. flour @ $6?
23.	*260 doz. eggs*	“ “ 78 yd. silk @ 90¢?

Factors and Multiples.

119. An integral (or whole) number of times a number is a multiple of that number. (See note, p. 53.)

Note.—A multiple of a number being some *whole times* that number, is, of course, always divisible by it; hence a multiple of a number is sometimes defined to be "a number that is exactly divisible by it."

SLATE EXERCISES.

1. Write in columns the multiples from 1 to 120 of 3, of 4, of 5, of 6, of 7, and of 8. Thus,

3	*4*	*5*	*6*	*7*	*8*
6	8	10	12	14	16
9	12	15	18	21	24
12	16	20	24	28	32
15	20	25	30	35	40
18	24	30	36	42	48
etc.	etc.	etc.	etc.	etc.	etc.

2. Make a list of the numbers from 1 to 120 that are multiples of both 3 and 4; thus—

12, 24, 36, 48, 60, 72, 84, 96, 108, 120.

3. In like manner make a list of the multiples from 1 to 120 that are common to 3 and 5, 3 and 6, etc.

4. Make a list of the multiples from 1 to 120 that are common to 3, 4 and 5; 3, 5 and 6, etc., as far as may be directed.

5. Make a list of the multiples that are common to 3, 4, 5 and 6; to 4, 5, 6 and 7; to 5, 6, 7 and 8, as above.

Note.—The pupil should note the fact that any number may have an unlimited number of multiples, and that any two or more numbers may have an unlimited number of *common* multiples, but only one *least common* multiple.

120. *Any multiple of a number must contain at least all the prime factors of that number.*

Thus 2 and 3 being factors of 6, they must occur as factors in any number of times 6. [3 times 6 is 3 times (2 times 3), and so on, to any number of times 6.] Though the line of marks at the right should be copied millions of times, the *number* of marks could never escape the factors 2 and 3. \\ \\ \\

121. *A multiple that is common to two or more numbers must contain at least all the prime factors that enter into each of them.*

Thus, any multiple common to 15 and 21 must contain the factors 3, 5 and 7, for no number that does not contain the factors 3 and 5 can be a multiple of 15, and no number that does not contain the factors 3 and 7 can be a multiple of 21.

Note.—A common multiple of 15 and 21 *may* contain any other factors besides 3, 5, and 7, but these it *must* contain.

122. *The least common multiple of two or more numbers must contain all the prime factors that enter into each of them, and no others.*

Thus, 18 and 24 being resolved (separated) into their prime factors, we have

$$18=2\times3\times3$$
$$24=2\times2\times2\times3.$$

Hence, any common multiple of 18 and 24 must contain the factors 2, 2, 2, 3 and 3, and the *least common multiple* must contain no other. If it did contain any other factor it would not be the LEAST common multiple.

ORAL EXERCISES

1. What is the least common multiple of 9, 14, and 21 ?

Oral Solution.—1. A multiple of 21 must contain the factors 3 and 7. 3 × 7=***21.***

2. A multiple of 14 must have the factors 2 and 7, hence the common multiple of 21 and 14 must contain 3, 7, and 2 as factors. 3 × 7 × 2=***42.***

3. A multiple of 9 must have the factors 3 and 3, hence the common multiple of 21, 14, and 9 must have the factors 3, 7, 2, and 3. 3 × 7 × 2 × 3=***126.***

Hence ***126*** is the *least common multiple* of 9, 14, and 21, because it contains no factor which is not necessary for one or another of the given numbers.

2. Find the least common multiple of 6, 7, 9, 12, 14, 18, 21, 36, and 42.

Oral Solution.—1. Since 36 is a multiple of 6, 9, 12, and 18, any multiple of 36 will be a multiple of these numbers also; and since 42 is a multiple of 7, 14, and 21, any multiple of 42 will be a multiple also of these numbers; hence, the least common multiple of 36 and 42 will be the least common multiple of all the given numbers.

2. The factors of 42 are 2, 3 and 7, but the factors of 36 are 2, 2, 3 and 3, or one 2 and one 3, more than are found in 42; hence we multiply 42 by 2 and by 3, or at once by 6, and obtain the product ***252***, which is the l. c. m. of all the given numbers.

Find the least common multiple

3. Of 2, 4, and 6
4. " 3, 4, and 6
5. " 5, 6, and 15
6. " 7, 14, and 21
7. Of 2, 3, 4, and 6
8. " 4, 8, 12, and 16
9. " 5, 7, 15, and 21
10. " 3, 14, 21, and 28
11. Of 9, 2, 6, 18, 24
12. " 8, 7, 12, 21, 24
13. " 5, 2, 15, 7, 35
14. " 2, 3, 6, 9, 54

Hence, for finding the least common multiple of two or more numbers, we have the following

123. ***Rule.*—1. Resolve each number into its prime factors.**

2. The continued product of the different prime factors, each used a number of times equal to the greatest number of times it occurs in any one of the given numbers will be the l. c. m.

Note.—If any of the given numbers are multiples of others, the multiples only need to be considered, for the given number that is a multiple of another contains all its factors.

SLATE EXERCISES.

15. Find the least common multiple of 88, 126, and 330.

Solution.

$88=2\times2\times2\times11$
$126=3\times3\times2\times7$
$330=2\times3\times5\times11$

Least Common Multiple.

$2\times3\times5\times11\times3\times7\times2\times2=27{,}720$

Explanation.—We write out all the factors of the several numbers, so that we may readily see what they are. We then take the factors of 330, and unite with them the factors that occur in the other numbers, and not in 330. The continued product of all is the least common multiple sought.

Note.—Any of the given numbers may be taken at once as the product of its own prime factors, and this being multiplied successively by such of the prime factors of each of the other numbers as are not contained in any preceding number. the product will be the l. c. m. sought. Thus:

$$330\times3\times7\times2\times2=27{,}720$$

In like manner find the l. c. m.

16. Of 27, 24, and 15
17. " 63, 27, and 84
18. " 12, 51, and 68
19. " 35, 63, and 72
20. Of 9, 12, 14, and 210
21. " 60, 15, 24, and 25
22. " 54, 81, 63, and 14
23. " 18, 24, 72, and 144
24. 19, 27, 36, 63
25. 13, 17, 19, 32
26. 23, 27, 54, 108
27. 14, 17, 105, 110

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28. Of 9546, 6364, and 14319
29. Of 4862, 2002, and 17017

Problems G. C. D. and L. C. M.

1. One boy's blocks are 2 inches thick, another's 3, and another's 5. The three boys build "towers" of equal heights. How high at least are they? How many blocks does each one use?

2. What is the least sum that can be paid in either 2, 3, 5, 10, 20, or 25¢ pieces?

3. William has 27¢, Mary 36¢, and Harry 51¢, not in one-cent pieces, yet all in coins of one denomination. What is it?

4. In one grammar school there are 504 girls, in another there are 324 boys. It is desired to divide them into classes of equal size. How many pupils will there be in each class, if as large as it can be made?

5. The four sides of a play-ground measure, respectively, 464, 672, 368, and 240 ft. in length. How long must the boards used in fencing it be cut, so that they shall be of equal length, and as long as possible?

6. On the same day a merchant sends out traveling salesmen with instructions to return, respectively, in 1, 2, 3, 4, and 5 weeks. When any one returns he is sent out again at once for the same period as before. In how many weeks will they be together again?

7. In how many weeks would they come in together if sent out for 6, 9, 12, 18, and 36 weeks, respectively?

8. What is the least sum a dealer in live stock must have to be able to invest equal sums in horses at $105, mules at $68, and beeves at $30 per head? How much if he pays $105 per head for horses, $70 for mules, and $30 for beeves?

9. A court-yard 42 ft. 6 in. long, and 31 ft. 8 in. wide, is to be paved with square tiles of equal size, and as large as possible. How long and wide must each tile be?

10. A man having on deposit $695, $417, and ~~$1251~~, respectively, in three different banks, wishes to draw out the whole in as large equal sums as possible. What is the greatest sum for which *he must draw* his checks?

CHAPTER IX.

FRACTIONS.

Introductory Exercises.

124. Draw on slate or paper twelve lines of equal length, and about one half inch apart. Divide and subdivide the lines as required by the questions.*

1. If the first line were divided into two equal parts, what would you call each part? How many such parts in a line? How many in 2, 5, 7, 9, 12 lines?

2. How many halves in 2 lines and a half? In 3 lines? In 4 lines and a half? In 11 lines and a half?

3. If each half were divided into two equal parts, how many of the new parts would there be in a whole line? What would you call one part? Two parts? etc. What part of a half is a fourth?

4. If the other lines were divided in the same way, how many fourths in each line? In 3 lines? In 8 lines? etc.

5. How many fourths in 2 lines and 1 fourth? In 5 and 1 fourth? In 6 and 3 fourths? In 7 and 1 fourth? etc.

6. Are 2 halves less or greater than a line? 5 fourths? etc.

7. How many whole lines in 2 halves? 4 halves? etc. How many in 3 fourths? 5 fourths? 9 fourths? etc.

8. If each fourth were divided into two equal parts, how many parts would there be in a line? What would you call them?

(Other questions should here be asked, similar to those on fourths, as above.)

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* Slips of paper of any uniform length, paper squares, etc., etc., are convenient materials for these exercises.

Draw 12 other lines. It would be well if these could be just 12 inches long.

1. If each line were divided into three equal parts, what would the parts be called? Why?

2. How would you express 1 part in figures? 2 parts? 5 parts?

(For reading and writing simple fractions see Art. 73, p. 77.)

3. Are $^{4}/_{3}$ greater or less than a whole line? How much? How many thirds in 3, 5, 7, 12 lines?

4. How many thirds in $2^{1}/_{3}$, $4^{1}/_{3}$, $5^{2}/_{3}$, $7^{1}/_{3}$, $8^{2}/_{3}$, $10^{3}/_{3}$?

5. How many whole lines, or how many lines and what parts of a line, are needed to make $^{7}/_{3}$, $^{9}/_{3}$, $^{14}/_{3}$, $^{21}/_{3}$, $^{15}/_{3}$, $^{30}/_{3}$, $^{32}/_{3}$?

6. If each third were divided into two equal parts, how many of the new parts would there be in a whole line? What would you call them? How would you express five of them in figures?

7. How many of these smaller parts are there in a third? In 2 thirds? In 3 thirds? How many sixths in 3 thirds?

8. Which is greater, $^{2}/_{3}$ or $^{2}/_{6}$ of one of these lines? Why? What part of a third is 1 sixth? Is $^{2}/_{6}$ a *part* or the whole of $^{1}/_{3}$?

9. How many sixths in $^{1}/_{3}$ of a line? In $^{1}/_{2}$? In $^{2}/_{3}$? In 2 lines? In 6, 8, 10, 11 lines?

10. How many sixths in $1^{1}/_{3}$? In $2^{5}/_{6}$? In $4^{1}/_{3}$? In $7^{5}/_{6}$? In $8^{4}/_{6}$? In $8^{2}/_{3}$? In $7^{2}/_{3}$?

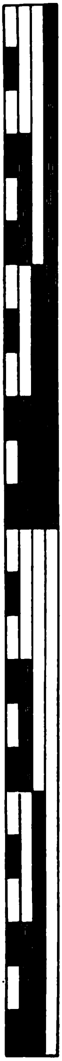
11. How many lines, or lines and parts of a line, are needed to make $^{18}/_{6}$, $^{21}/_{6}$, $^{24}/_{6}$, $^{36}/_{6}$, $^{39}/_{6}$, $^{42}/_{6}$, $^{49}/_{6}$, $^{54}/_{6}$, $^{58}/_{6}$?

12. If each sixth were divided into two equal parts, what would the new parts be called? How would you express one or more of them? If the *line* is 12 inches long, what is the length of each *part?*

13. How many of them in $^{1}/_{3}$ of a line? In $^{2}/_{3}$, $^{4}/_{6}$, $^{1}/_{6}$, $^{1}/_{3}$, $^{3}/_{6}$?

14. How many twelfths in 2 lines and $^{1}/_{2}$? In $3^{3}/_{12}$? In $4^{5}/_{12}$? In $7^{1}/_{12}$? In $10^{7}/_{12}$? In $11^{11}/_{12}$? How many twelfths in $1^{1}/_{3}$, $2^{1}/_{6}$, $4^{5}/_{6}$, $7^{3}/_{12}$, $9^{2}/_{3}$ lines?

Questions upon the Rules in the Margin.



125. **Note.**—The following questions are designed to be only suggestive of exercises that may be given. A foot-rule or a yard-stick will afford many others.

1. Into how many parts is the first of these two measures divided by the horizontal line in the middle ? What do you call the parts ? Into how many parts is the measure at the right divided by the longest horizontal lines ? What do you call these parts ? Why ?

2. Which is greater, $^1/_2$ or $^1/_3$? How can you tell without seeing or measuring the parts ?

3. What parts of the whole do you get by dividing $^1/_3$ into 2 equal parts ? Why ?

4. How many sixths in $^1/_3$, $^2/_3$, $^3/_3$? What part of $^1/_2$ do you get by dividing it into 2 equal parts? What part of the whole is $^1/_2$ of $^1/_2$?

5. How many parts do you get by dividing each of the fourths into 2 equal parts ? What are these new parts called ? Why ?

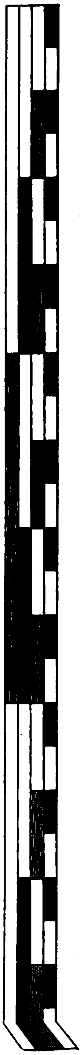
6. Which is longer, $^1/_6$ or $^1/_8$? Suppose you could not see, nor measure, would you know which is the greater, $^3/_6$ or $^3/_8$? How ?

7. The sixths in the second measure are divided each into 2 equal parts. What is their name ? Why ?

8. Are these twelfths as large as the eighths in the other measure ?

9. What are the smallest parts of the second measure ? How many are there ?

10. Are the smallest parts of the first measure as large as the smallest parts of the second measure ? Can you tell by counting them ?



ORAL EXERCISES.

1. Explain how it is that $^1/_2$ is equal to $^2/_4$.

Note.—Divide any whole thing or number into fourths, and show that one half is equal to 2 fourths.

2. Is $^1/_2$ equal to $^3/_6$, to $^4/_8$, to $^6/_{12}$, to $^{12}/_{24}$? State why.

3. Name some other parts equal to $^1/_4$; also parts equal to $^1/_6$, to $^1/_8$, to $^1/_3$, to $^1/_{12}$, to $^2/_3$, to $^3/_4$, to $^5/_8$, to $^5/_6$, to $^7/_8$, to $^3/_8$.

4. If you had a line divided into sixths, how could you change the sixths into twelfths? The twelfths back to sixths?

5. How many sixths in $^1/_3$, $^2/_3$? How many eighths in $^3/_4$, $^1/_2$?

6. How many twelfths in $^1/_6$, $^1/_3$, $^1/_4$, $^1/_2$? In $^2/_2$, $^2/_3$, $^2/_4$, $^2/_6$?

7. How many twenty-fourths in $^3/_8$? In $^5/_8$? In $^7/_8$? How many in $^2/_6$, $^4/_6$, $^3/_6$, $^5/_6$? How many in $^1/_4$, $^2/_4$, $^3/_4$?

How much is

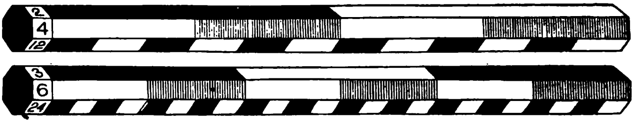
8. $^1/_2$ of $^1/_2$?
$^1/_2$ of $^1/_4$?
$^1/_2$ of $^1/_3$?

9. $^1/_2$ of $^1/_8$?
$^1/_2$ of $^1/_6$?
$^1/_2$ of $^1/_{12}$?

10. $^1/_4$ of $^1/_2$?
$^1/_4$ of $^1/_3$?
$^1/_3$ of $^1/_4$?

11. $^1/_3$ of $^1/_2$?
$^1/_8$ of $^1/_3$?
$^1/_3$ of $^1/_3$?

Note.—The following representation of a fraction rule will suggest other exercises. The figures at the left show how many parts each side is divided into.



12. How many wholes and ninths are in $^{15}/_9$, $^{29}/_9$, $^{21}/_9$, $^{30}/_9$, $^{47}/_9$?

13. Which makes the larger parts, dividing an apple into 10ths or 12ths? Which is the greater, $^1/_8$ or $^1/_{12}$ of a thing? $^1/_{20}$ or $^1/_{16}$? $^1/_{30}$ or $^1/_{31}$? $^1/_4$ or $^1/_9$?

14. Which is the greater, $^7/_{24}$ or $^{17}/_{24}$? $^4/_7$ or $^6/_7$? $^{73}/_{145}$ or $^{29}/_{145}$? $^5/_8$ or $^3/_8$? $^7/_{18}$ or $^{10}/_{18}$? Why?

15. Draw two lines of equal length. Divide one into thirds, the other into fourths, and find how many more twelfths there are in $^1/_3$ than in $^1/_4$.

Definitions.

126. A ***Fraction*** is one or more of the equal parts of a unit or whole.

127. The ***unit of the fraction*** is the unit which is divided. *One* of the equal parts is a ***fractional unit.***

128. Fractions obtained by the division of the unit into tenths, tenths of tenths or hundredths, etc., are called ***Decimal Fractions.*** All others are called ***Common Fractions,*** to distinguish them from decimals.

129. **Common Fractions** may be expressed by words, as *two thirds,* or by figures, thus, $^2/_3$, the upper number standing for "two," the *number* of parts, and the lower one for "thirds," the *name* of the parts. (See Art. 73, page 77.)

130. The *number* of parts and the *name* of the parts are called the ***terms*** of the fraction.

131. The term which expresses the number of parts is the ***numerator*** (counter or numberer). The term which indicates the name of the parts is the ***denominator*** (namer).

Note.—Since the denominator indicates the name of the parts by showing *how many* parts there are in a unit, it may be treated as a number as well as a name.

132. A ***simple fraction*** is one whose terms are both integers, as $^9/_9$, $^{17}/_{20}$, etc.

133. A ***proper fraction*** is one whose numerator is less than the denominator, as $^2/_3$, $^3/_4$, etc.

134. An ***improper fraction*** is one whose numerator is equal to or greater than its denominator, as $^6/_6$, $^8/_7$, etc.

135. A ***mixed number*** is one which is composed of an integer and a fraction, as $3\,^1/_2$, $5\,^3/_7$, etc.

136. An integer may be expressed in the form of an improper fraction by writing it as a numerator, with 1 as a denominator. **Thus, 5 may be written $^5/_1$, which is read 5 *ones* or 5.**

Reductions.

Changes of Form, not of Value.

137. To reduce integers or mixed numbers to improper fractions, and the contrary.

Example.—1. Reduce $67^{3}/_{5}$ to fifths.

Analysis.—In 1 there are 5 fifths, hence in 67 there are 67 times 5 fifths = 335 fifths; 335 fifths + 3 fifths = 338 fifths.

Note.—The result being the same, we multiply 67 by 5, as the shortest way of obtaining 67 times 5.

Slate Work.

$$\begin{array}{r} 67 \\ \underline{5} \\ 335 \\ \underline{3} \\ 338 \end{array}$$

Example.—2. Reduce $^{19}/_{5}$ to an integer or mixed number.

Analysis.—5 fifths = 1. Hence 19 fifths contain as many units as there are times 5 fifths in 19 fifths = $3^{4}/_{5}$.

Suggestion.—For the rules in these cases the pupil may be required to state the processes by which he obtains the results.

Slate Work.

$$\begin{array}{r} 5\underline{)19} \\ 3^{4}/_{5} \end{array}$$

Note.—In oral exercises the pupil should be required to announce results at once if possible, except when specially directed to give an analysis.

Reduce to improper fractions

3.	**4.**	**5.**	**6.**	**7.**	**8.**
$1^{3}/_{5}$	$2^{2}/_{5}$	$5^{7}/_{8}$	$4^{1}/_{3}$	$19^{2}/_{3}$	$31^{1}/_{4}$
$7^{1}/_{8}$	$1^{7}/_{8}$	$8^{5}/_{7}$	$3^{3}/_{5}$	$18^{3}/_{4}$	$29^{1}/_{2}$
$5^{1}/_{2}$	$3^{1}/_{7}$	$7^{5}/_{8}$	$8^{2}/_{7}$	$37^{1}/_{2}$	$33^{1}/_{3}$

Reduce to integers or mixed numbers

9.	**10.**	**11.**	**12.**	**13.**	**14.**
$^{5}/_{3}$	$^{11}/_{8}$	$^{16}/_{2}$	$^{23}/_{7}$	$^{39}/_{8}$	$^{33}/_{2}$
$^{3}/_{2}$	$^{9}/_{7}$	$^{15}/_{3}$	$^{25}/_{3}$	$^{55}/_{7}$	$^{44}/_{2}$
$^{5}/_{4}$	$^{8}/_{3}$	$^{17}/_{5}$	$^{32}/_{7}$	$^{53}/_{9}$	$^{39}/_{3}$

Reduce mixed numbers to improper fractions, and improper fractions to integers or mixed numbers.

15. $28^{7}/_{16}$
16. $31^{6}/_{13}$
17. $24^{8}/_{17}$
18. $^{421}/_{7}$
19. $^{357}/_{6}$
20. $^{216}/_{5}$
21. $100^{10}/_{11}$
22. $375^{10}/_{23}$
23. $841\,^{5}/_{51}$
24. $^{1828}/_{14}$
25. $^{1327}/_{13}$
26. $^{4563}/_{15}$
27. $487^{5}/_{6}$
28. $723^{9}/_{10}$
29. $891^{9}/_{12}$

30. How many yards in $^{356}/_{9}$ of a yard?

Reducing to Higher and Lower Terms.

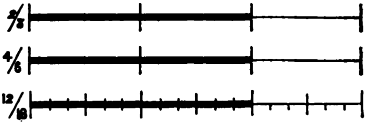
Let it be remembered that *the value of a fraction is not changed by multiplying or dividing both terms by the same number; thus—*

$$\frac{4\times3}{6\times3}=\frac{12}{18} \qquad \frac{4\div2}{6\div2}=\frac{2}{3}$$

For it is clear that

is equivalent to

or to



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138. To reduce a fraction to higher terms (greater numerator and denominator).

Example.—1. Reduce $^3/_4$ to twelfths.

Process.

$$\frac{3\times3}{4\times3}=\frac{9}{12}$$

Explanation.—If each fourth of a slip of paper be divided into three equal parts, the whole slip will contain 4 times 3 parts, or 12 twelfths, and 3 fourths will contain 3 times 3 parts, or $^9/_{12}$.

Hence the following

Rule.—**To reduce a fraction to higher terms, divide the required denominator by the denominator of the given fraction, and multiply both of its terms by the quotient.**

ORAL EXERCISES.

Reduce

2. $^1/_2$, $^1/_4$, $^1/_6$, $^1/_3$, to 12ths.
3. 1, $^1/_4$, $^3/_8$, $^1/_2$, " 8ths.
4. $^1/_2$, $^1/_3$, $^1/_6$, $^5/_9$, " 18ths.
5. $^2/_3$, $^3/_4$, $^5/_6$, $^7/_8$, " 24ths.
6. $^3/_4$, $^5/_8$, $^1/_2$, $^5/_{16}$, " 16ths.

Reduce

7. $^2/_3$, $^5/_6$, $^7/_8$, $^7/_{12}$, $^5/_{18}$, to 72ds.
8. $^1/_2$, $^5/_6$, $^4/_9$, $^{14}/_{27}$, $^{14}/_{18}$, " 54ths.
9. $^2/_3$, $^3/_5$, $^7/_9$, $^5/_9$, $^8/_{15}$, " 45ths.
10. $^5/_8$, $^1/_4$, $^9/_{16}$, $^1/_2$, $^{17}/_{24}$, " 48ths.
11. $^7/_9$, $^3/_4$, $^7/_{12}$, $^{11}/_3$, $^{17}/_{18}$, " 36ths.

Let the first three examples be illustrated by division of lines or folding of paper.

12. Change $^1/_2$, $^1/_5$, $^2/_5$, $^3/_5$, $^4/_5$, and $^5/_5$ to tenths.

13. Change to hundredths $^1/_4$, $^2/_5$, $^7/_{10}$, $^3/_{20}$, $^3/_5$, $^7/_{25}$, $^9/_{10}$, $^4/_5$, $^3/_4$, $^{23}/_{25}$

139. To reduce fractions to lower terms (smaller numerator and denominator).

Note.—In the preceding case (p. 141) we computed the numerical result of dividing any given equal parts of a thing or number into smaller equal parts. In this case, we are to find the result of uniting smaller into larger ones.

Example.—1. Reduce $^9/_{12}$ to lower terms.

Process.

$$3\left|\frac{9}{12}\right. = {}^3/_4$$

Explanation.—Uniting each 3 twelfths of any object into 1 larger part, we have 4 larger parts (fourths), and in the 9 twelfths there are 3 of them. Hence $^9/_{12} = {}^3/_4$.

Reduce to lower terms

2. $^{12}/_{24}$ $^{64}/_{72}$ $^5/_{90}$ $^{66}/_{240}$ $^{59}/_{177}$ $^{10}/_{50}$ $^{48}/_{72}$ $^{15}/_{75}$ $^{64}/_{144}$ $^{41}/_{164}$

3. $^{15}/_{35}$ $^6/_{18}$ $^{12}/_{96}$ $^{72}/_{144}$ $^{480}/_{500}$ $^{15}/_{45}$ $^{31}/_{93}$ $^{34}/_{68}$ $^{48}/_{144}$ $^{196}/_{444}$

140. Thus we find that *fractions are reduced to lower terms by dividing both terms by any common factor.*

And, *that they are reduced to their lowest terms by dividing them, successively, by* ***all*** *the prime factors common to the two;* or, *by the continued product of all, the latter being their greatest common factor, and hence their greatest common divisor.*

141. When the terms of a fraction are large, or not readily resolved into factors, the following method of finding the greatest common divisor will be convenient.

4. Reduce $^{475}/_{589}$ to lowest terms.

Process.

```
      475)589(1
          475
          ---
          114)475(4
              456
              ---
               19)114(6
                  114
```

$$19\left|\frac{475}{589}\right. = {}^{25}/_{31}$$

Explanation.—We divide the greater number by the less, and the divisor by the remainder, and so on till there is no remainder. The last divisor (19) is the g. c. d. sought.

Why it is that we can always depend on such a process to find the g. c. d. is not readily understood by the young learner. The demonstration is therefore reserved for the appendix. No formal rule is necessary.

Reduce to lowest terms

5. $^{1645}/_{1833}$ **7.** $^{1363}/_{1739}$ **9.** $^{8903}/_{13201}$ **11.** $^{1261}/_{1649}$ **13.** $^{1989}/_{2873}$

6. $^{1589}/_{2724}$ **8.** $^{8903}/_{10991}$ **10.** $^{1945}/_{3501}$ **12.** $^{2813}/_{3763}$ **14.** $^{1945}/_{3501}$

Addition of Common Fractions.

ORAL EXERCISES.

1. Mary takes $^1/_6$ of a pie for lunch at school, William takes $^2/_6$, John $^2/_6$, and Henry $^2/_6$. How many sixths do they all take ?

Find the sum of

2. $^1/_5 + ^3/_5 =$ **3.** $^3/_4 + ^1/_4 =$ **4.** $^5/_{11} + ^8/_{11} =$ **5.** $^4/_5 + ^3/_5 + ^2/_5 =$
$^4/_7 + ^2/_7 =$ $^6/_{10} + ^4/_{10} =$ $^6/_{12} + ^7/_{12} =$ $^7/_8 + ^5/_8 + ^3/_8 =$
$^3/_8 + ^4/_8 =$ $^7/_{13} + ^6/_{13} =$ $^9/_{17} + ^9/_{17} =$ $^4/_9 + ^5/_9 + ^7/_9 =$

6. Sarah has $^5/_8$ of a yard of ribbon, and Lucy has $^7/_8$; how many 8ths have they together ? How many yards, and what part of a yard ?

Find the sum of

7. $7\,^1/_2 + ^1/_2 =$ **8.** $5\,^6/_7 + ^3/_7 =$ **9.** $6\,^5/_6 + 3\,^1/_6 =$ **10.** $16\,^7/_{16} + ^9/_{16} =$
$3\,^2/_5 + ^3/_5 =$ $4\,^3/_8 + ^7/_8 =$ $3\,^9/_{11} + 5\,^2/_{11} =$ $14\,^{30}/_{59} + ^{35}/_{59} =$
$5\,^4/_7 + ^3/_7 =$ $8\,^4/_9 + ^7/_9 =$ $5\,^6/_{17} + 8\,^{11}/_{17} =$ $12\,^{17}/_{35} + ^{19}/_{35} =$

11. One piece of cloth contains $^3/_4$ of a yard, and another $^2/_3$ of a yard. How many yards in the two pieces ?

Oral Solution.—$^3/_4$ is equal to $^9/_{12}$, $^2/_3$ is equal to $^8/_{12}$, $^9/_{12}$ and $^8/_{12}$ together are equal to $^{17}/_{12}$. $^{17}/_{12} = 1\,^5/_{12}$; hence, $^3/_4$ and $^2/_3$ of a yard of cloth equal $1\,^5/_{12}$ yd.

Illustration.—3 fourths and 2 thirds of a sheet of paper make neither 5 fourths nor 5 thirds, but subdividing both into twelfths we find that they are together equal to $^{17}/_{12}$ or $1\,^5/_{12}$.

142. *Fractions to be added together must have a common denominator.*

Find the sum of

12. $^1/_2 + ^1/_3 =$ **13.** $^1/_2 + ^3/_4 =$ **14.** $^2/_3 + ^2/_6 =$ **15.** $^5/_6 + ^7/_{12} =$
$^1/_4 + ^1/_2 =$ $^1/_4 + ^3/_8 =$ $^1/_2 + ^4/_5 =$ $^3/_8 + ^1/_{24} =$
$^1/_3 + ^1/_6 =$ $^1/_3 + ^5/_6 =$ $^3/_4 + ^7/_8 =$ $^5/_7 + ^5/_{14} =$
$^1/_4 + ^1/_8 =$ $^1/_2 + ^3/_5 =$ $^2/_3 + ^5/_6 =$ $^2/_3 + ^7/_{15} =$
$^2/_3 + ^4/_5 =$ $^4/_5 + ^5/_6 =$ $^2/_3 + ^7/_8 =$ $^3/_4 + ^3/_5 =$

16. $^1/_2 + ^1/_4 + ^1/_8 =$ **18.** $^1/_2 + ^1/_3 + ^1/_4 + ^1/_6 =$ **20.** $^1/_2 + ^1/_3 + ^1/_4 + ^1/_6 + ^1/_8 =$
17. $^2/_3 + ^5/_6 + ^6/_{12} =$ **19.** $^1/_2 + ^4/_5 + ^3/_{10} + ^7/_{15} =$ **21.** $^6|_7 + ^2|_3 + ^{11}|_{21} =$

WRITTEN WORK.

143. Example.—1. Find the sum of $^{7}/_{18}$ and $^{8}/_{15}$.

Solution with Sheets of Paper.—By subdividing 18ths and 15ths each into 2, 3, etc., equal parts,

The 18ths become successively 36ths, 54ths, 72ds, 90ths, 108ths, etc.

The 15ths become successively 30ths, 45ths, 60ths, 75ths, 90ths, etc.

We thus find that 18ths and 15ths can both be changed to 90ths by dividing each 18th into 5, and each 15th into 6 equal parts; and hence that the common denominator of the equivalent fractions must be 90, which is the l. c. m. of 18 and 15.

The arithmetical process of finding the sum of two or more fractions having unlike denominators embraces corresponding steps, viz., *first*, finding the l. c. m. of the denominators; *second*, reducing the given fractions to a common denominator; and, *third*, adding together the numerators. Thus,

The Arithmetical Process.

$$\begin{array}{lr|l}
18=2\times3\times3 & & 90 \\
15=5\times3 & 18 & 5\times7=35 \\
 & 15 & 6\times8=48 \\
\text{Least Com. Mult.} & & \quad ^{83}/_{90} \\
2\times3\times3\times5=90 & &
\end{array}$$

Explanation.—Having found the l. c. m. we divide it by 18 and 15, and thus find that we must multiply the 7 (eighteenths) by 5, and the 8 (fifteenths) by 6 to change them to 90ths. Having performed these multiplications we add $^{35}/_{90}$ and $^{48}/_{90}$ to obtain the sum $^{83}/_{90}$.

By dispensing with such part of the written work as can be performed without the aid of the pencil, it may be abbreviated as follows:

2. Find the sum of $^{3}/_{4}$, $^{1}/_{8}$, $^{5}/_{6}$, $^{1}/_{12}$, $^{2}/_{3}$.

$$\begin{array}{r|l}
 & 24\text{ths} \\
\hline
^{3}/_{4} & 18 \\
^{1}/_{8} & 3 \\
^{5}/_{6} & 20 \\
^{1}/_{12} & 2 \\
^{2}/_{3} & 16 \\
\hline
 & ^{59}/_{24}=2\,^{11}/_{24}
\end{array}$$

Explanation.—24 being the least common multiple of the denominators, the given fractions may all be reduced to 24ths. In $^{1}/_{4}$ there are $^{6}/_{24}$, and in $^{3}/_{4}$ there are 3 times 6 or $^{18}/_{24}$, etc., etc.

144. *Rule.*—1. Reduce the fractions, if necessary, to fractions having a common denominator.

2. Add the numerators, and under the sum write the common denominator.

3. In adding mixed numbers, add first the fractions, then the integers, and unite the results.

Caution.—If any arithmetical process results in a fraction, **the work is not complete** unless the fraction is expressed in its lowest terms. **Improper fractions** ***must be reduced*** to whole or mixed numbers.

Add by columns, then by lines. Test results.[*] (See note, p. 32.)

3. **4.** **5.** **6.** **7.** **8.**

9. $^{2}/_{5} + ^{2}/_{3} + ^{1}/_{2} + ^{5}/_{6} + ^{11}/_{12} + ^{7}/_{24}$

10. $^{3}/_{4} + ^{1}/_{4} + ^{7}/_{12} + ^{1}/_{2} + ^{7}/_{8} + ^{9}/_{16}$

11. $^{7}/_{8} + ^{2}/_{3} + ^{11}/_{12} + ^{3}/_{4} + ^{17}/_{24} + ^{4}/_{5}$

12. $^{5}/_{6} + ^{7}/_{12} + ^{5}/_{14} + ^{2}/_{3} + ^{3}/_{4} + ^{1}/_{2}$

13. **14.** **15.** **16.** **17.** **18.**

19. $^{1}/_{3} + ^{3}/_{4} + ^{2}/_{3} + ^{11}/_{12} + ^{3}/_{7} + ^{5}/_{6}$

20. $^{5}/_{8} + ^{6}/_{7} + ^{5}/_{6} + ^{3}/_{5} + ^{7}/_{9} + ^{3}/_{4}$

21 $^{3}/_{7} + ^{5}/_{8} + ^{6}/_{11} + ^{5}/_{9} + ^{1}/_{2} + ^{1}/_{6}$

22. $^{5}/_{6} + ^{1}/_{7} + ^{7}/_{8} + ^{9}/_{11} + ^{3}/_{4} + ^{2}/_{5}$

23. $24\,^{7}/_{16} + 18\,^{3}/_{5} + 50\,^{3}/_{10} + 18\,^{7}/_{30} + 4\,^{1}/_{2} + 2\,^{5}/_{6} + 59\,^{3}/_{7}$.

24. $3\,^{4}/_{9} + 5\,^{5}/_{8} + 17\,^{1}/_{6} + 4\,^{5}/_{72} + 37\,^{13}/_{24} + 17\,^{11}/_{12} + 70\,^{5}/_{9}$.

25. $15\,^{7}/_{9} + 24\,^{3}/_{4} + 38\,^{3}/_{5} + 27\,^{13}/_{16} + 33\,^{14}/_{25} + 19\,^{7}/_{25} + 8\,^{1}/_{8}$.

26. **27.** **28.** **29.** **30.** **31.**

32. $4\,^{4}/_{5} + 3\,^{1}/_{5} + 4\,^{2}/_{4} + 13\,^{1}/_{2} + 17\,^{1}/_{60} + 14\,^{7}/_{24}$

33. $63\,^{3}/_{4} + 21\,^{7}/_{20} + 45\,^{7}/_{20} + 25\,^{3}/_{4} + 14\,^{23}/_{30} + 22\,^{5}/_{12}$

34. $227\,^{7}/_{10} + 243\,^{3}/_{4} + 26\,^{9}/_{10} + 36\,^{7}/_{8} + 11\,^{11}/_{15} + 46\,^{3}/_{8}$

35. $438\,^{39}/_{50} + 657\,^{43}/_{50} + 38\,^{27}/_{50} + 49\,^{15}/_{16} + 19\,^{11}/_{12} + 39\,^{5}/_{6}$

36. $840\,^{1}/_{2} + 760\,^{27}/_{100} + 14\,^{99}/_{100} + 51\,^{5}/_{8} + 15\,^{5}/_{6} + 10\,^{1}/_{2}$

37. Add $^{2}/_{3} + ^{5}/_{8} + ^{4}/_{9} + ^{2}/_{5} + ^{3}/_{4} + ^{7}/_{16} + ^{4}/_{7} + ^{3}/_{16} + ^{4}/_{15}$

38. Add $^{3}/_{4} + ^{4}/_{5} + ^{11}/_{12} + ^{3}/_{8} + ^{4}/_{15} + ^{5}/_{9} + ^{5}/_{7} + ^{4}/_{15} + ^{7}/_{18}$

39. Add $^{4}/_{5} + ^{5}/_{6} + ^{7}/_{8} + ^{4}/_{9} + ^{5}/_{12} + ^{23}/_{48} + ^{17}/_{60} + ^{7}/_{48} + ^{1}/_{24}$

Applications.—1. Four barrels of cider contain severally $25\,^{3}/_{4}$ gal., $23\,^{7}/_{12}$ gal., $29\,^{5}/_{6}$ gal., $28\,^{5}/_{6}$ gal. How many gallons in all?

2. What is the total weight of 6 bales, weighing respectively $5\,^{3}/_{8}$ cwt., $4\,^{7}/_{10}$ cwt., $6\,^{3}/_{5}$ cwt., $4\,^{7}/_{8}$ cwt., $6\,^{19}/_{20}$ cwt., $6\,^{1}/_{2}$ cwt.?

3. Mr. Abel has $64\,^{7}/_{20}$ acres in farm-land, $35\,^{5}/_{8}$ in meadow-land, $32\,^{19}/_{50}$ in woodland. How many acres in all?

4. Of 5 brothers the youngest is $11\,^{2}/_{5}$ years old, the second $8\,^{1}/_{4}$ years older, the third $2\,^{1}/_{6}$ years older than the second, the fourth $2\,^{1}/_{5}$ years older than the third, the fifth $2\,^{7}/_{9}$ years older than the fourth. How old is each one?

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[*] Any two or more columns may be assigned for an exercise. Since these examples are self-testing, no answers are given.

Subtraction of Common Fractions.

ORAL EXERCISES.

1. If there are ${}^{5}/_{7}$ of a pie on a plate, and Harry takes ${}^{2}/_{7}$, what part of the pie remains ?

2.	3.	4.	5.	6.
${}^{7}/_{8} - {}^{5}/_{8} =$	${}^{8}/_{8} - {}^{3}/_{8} =$	$5 - {}^{1}/_{2} =$	$7 - {}^{3}/_{4} =$	$2\,{}^{5}/_{8} - {}^{3}/_{8} =$
${}^{17}/_{18} - {}^{11}/_{18} =$	${}^{7}/_{7} - {}^{2}/_{7} =$	$6 - {}^{1}/_{4} =$	$8 - {}^{5}/_{9} =$	$4\,{}^{9}/_{10} - {}^{7}/_{10} =$
${}^{21}/_{25} - {}^{12}/_{25} =$	${}^{9}/_{9} - {}^{4}/_{9} =$	$3 - {}^{2}/_{3} =$	$9 - {}^{2}/_{5} =$	$6\,{}^{4}/_{5} - {}^{2}/_{5} =$
${}^{81}/_{100} - {}^{25}/_{100} =$	${}^{6}/_{6} - {}^{2}/_{6} =$	$4 - {}^{2}/_{5} =$	$10 - {}^{3}/_{7} =$	$7\,{}^{6}/_{7} - {}^{3}/_{7} =$

7.	8.	9.	10.
$2\,{}^{3}/_{8} - {}^{5}/_{8} =$	$3\,{}^{3}/_{10} - {}^{7}/_{10} =$	$8\,{}^{1}/_{4} - 4\,{}^{3}/_{4} =$	$7\,{}^{3}/_{9} - 3\,{}^{7}/_{9} =$
$4\,{}^{5}/_{12} - {}^{10}/_{12} =$	$7\,{}^{7}/_{20} - {}^{11}/_{20} =$	$59\,{}^{2}/_{5} - 4\,{}^{4}/_{5} =$	$33\,{}^{2}/_{7} - 2\,{}^{6}/_{7} =$
$6\,{}^{5}/_{9} - {}^{7}/_{9} =$	$6\,{}^{4}/_{13} - {}^{5}/_{13} =$	$85\,{}^{1}/_{3} - 1\,{}^{2}/_{3} =$	$22\,{}^{3}/_{16} - 1\,{}^{15}/_{16} =$
$5\,{}^{5}/_{17} - {}^{8}/_{17} =$	$9\,{}^{5}/_{8} - {}^{7}/_{8} =$	$98\,{}^{7}/_{11} - 2\,{}^{5}/_{11} =$	$43\,{}^{4}/_{15} - 1\,{}^{14}/_{15} =$
$2\,{}^{4}/_{7} - {}^{6}/_{7} =$	$3\,{}^{6}/_{27} - {}^{7}/_{27} =$	$14\,{}^{3}/_{41} - 3\,{}^{5}/_{41} =$	$17\,{}^{19}/_{60} - 4\,{}^{28}/_{60} =$

11. Subtract the sum of ${}^{3}/_{44}$, ${}^{5}/_{44}$, ${}^{17}/_{44}$, ${}^{22}/_{44}$, and ${}^{13}/_{44}$, from $5\,{}^{1}/_{44}$.

12. Subtract the sum of ${}^{6}/_{19}$, ${}^{3}/_{19}$, ${}^{5}/_{19}$, ${}^{17}/_{19}$, and ${}^{15}/_{19}$, from $7\,{}^{9}/_{19}$.

13. Subtract the sum of $1\,{}^{3}/_{100}$, $4\,{}^{7}/_{100}$, $5\,{}^{13}/_{100}$, $6\,{}^{17}/_{100}$, and $2\,{}^{9}/_{100}$, from $49\,{}^{7}/_{100}$.

14. Sarah has ${}^{5}/_{6}$ of a yard of velvet. How much will she have left if she uses ${}^{3}/_{4}$ of a yard for trimming a dress ?

Oral Solution.—${}^{5}/_{6}$ is equal to ${}^{10}/_{12}$, ${}^{3}/_{4}$ is equal to ${}^{9}/_{12}$; ${}^{9}/_{12}$ being subtracted from ${}^{10}/_{12}$ leaves ${}^{1}/_{12}$. Hence, Sarah would have ${}^{1}/_{12}$ of a yard remaining.

145. *That one fraction may be subtracted from another, the two must have a common denominator.*

146. ***Rule.*—1. Reduce the fractions, if necessary, to fractions having a common denominator.**

2. Subtract the numerator of the subtrahend from the numerator of the minuend, and write the remainder over the common denominator.

3. In the subtraction of mixed numbers, if the fraction in the subtrahend is greater than that in the minuend, take 1 unit from the whole number in the minuend and add it in fractional form to the fraction of the minuend, and then subtract.

ORAL AND SLATE EXERCISES.

Note.—The oral exercises may be carried as far as the ability of the pupil will permit.

15.	**16.**	**17.**	**18.**	**19.**
$\frac{1}{2}-\frac{1}{4}=$	$\frac{1}{2}-\frac{3}{8}=$	$\frac{3}{4}-\frac{5}{8}=$	$1\frac{1}{4}-\frac{7}{8}=$	$3\frac{3}{4}-\frac{1}{2}=$
$\frac{1}{3}-\frac{1}{6}=$	$\frac{1}{3}-\frac{1}{12}=$	$\frac{2}{3}-\frac{5}{12}=$	$2\frac{1}{3}-\frac{8}{9}=$	$2\frac{7}{8}-\frac{3}{4}=$
$\frac{1}{4}-\frac{1}{8}=$	$\frac{1}{4}-\frac{5}{24}=$	$\frac{5}{6}-\frac{7}{12}=$	$4\frac{5}{9}-\frac{17}{18}=$	$7\frac{3}{6}-\frac{2}{3}=$
$\frac{1}{5}-\frac{1}{10}=$	$\frac{1}{8}-\frac{1}{16}=$	$\frac{4}{9}-\frac{5}{18}=$	$6\frac{3}{8}-\frac{7}{16}=$	$9\frac{1}{18}-\frac{5}{9}=$
$\frac{1}{2}-\frac{1}{8}=$	$\frac{1}{5}-\frac{2}{25}=$	$\frac{5}{8}-\frac{7}{16}=$	$7\frac{1}{12}-\frac{5}{24}=$	$7\frac{7}{12}-\frac{3}{4}=$
$\frac{1}{3}-\frac{1}{9}=$	$\frac{1}{7}-\frac{2}{21}=$	$\frac{9}{10}-\frac{11}{20}=$	$5\frac{3}{11}-\frac{5}{22}=$	$6\frac{5}{21}-\frac{2}{3}=$
$\frac{1}{8}-\frac{1}{24}=$	$\frac{1}{4}-\frac{3}{16}=$	$\frac{3}{8}-\frac{5}{24}=$	$8\frac{4}{25}-\frac{7}{50}=$	$5\frac{6}{25}-\frac{4}{5}=$

20.	**21.**	**22.**	**23.**
$4\frac{7}{8}-3\frac{5}{16}=$	$7\frac{4}{5}-2\frac{1}{3}=$	$6\frac{2}{3}-1\frac{1}{5}=$	$5\frac{3}{4}-3\frac{2}{3}=$
$3\frac{5}{6}-1\frac{3}{5}=$	$13\frac{5}{8}-1\frac{1}{6}=$	$24\frac{7}{9}-2\frac{3}{8}=$	$7\frac{1}{11}-3\frac{5}{12}=$
$2\frac{1}{6}-1\frac{7}{8}=$	$25\frac{8}{9}-2\frac{7}{8}=$	$37\frac{6}{11}-3\frac{7}{10}=$	$9\frac{1}{2}-5\frac{2}{27}=$

24. Subtract \$$2\frac{3}{4}$ from \$$7\frac{1}{2}$, \$$6\frac{1}{6}$, \$$5\frac{5}{8}$, \$$9\frac{3}{7}$, \$$4\frac{2}{9}$.

25. Subtract $1\frac{5}{8}$ lb. from $6\frac{1}{16}$, $7\frac{3}{8}$, $9\frac{5}{16}$, $11\frac{7}{24}$, $5\frac{5}{12}$ lb.

26. Subtract $3\frac{5}{6}$ qt. from $8\frac{1}{12}$, $9\frac{7}{18}$, $11\frac{5}{8}$, $7\frac{7}{30}$ qt.

27. $2\frac{7}{16}-1\frac{31}{48}=$
28. $3\frac{3}{20}-1\frac{17}{120}=$
29. $74\frac{7}{25}-2\frac{19}{50}=$
30. $68\frac{11}{20}-9\frac{7}{9}=$
31. $15\frac{13}{18}-9\frac{8}{9}=$
32. $60\frac{15}{16}-3\frac{1}{7}=$
33. $25\frac{13}{16}-2\frac{7}{12}=$
34. $56\frac{2}{9}-27\frac{5}{6}=$
35. $165\frac{3}{8}-39\frac{7}{12}=$
36. $283\frac{5}{6}-46\frac{3}{14}=$
37. $394\frac{4}{15}-53\frac{7}{10}=$
38. $443\frac{7}{20}-31\frac{9}{16}=$
39. $527\frac{6}{17}-13\frac{7}{12}=$
40. $136\frac{19}{24}-4\frac{3}{4}=$
41. $48\frac{3}{8}-7\frac{11}{12}=$
42. $824\frac{2}{5}-6\frac{7}{12}=$
43. $936\frac{4}{15}-4\frac{11}{20}=$
44. $141\frac{7}{20}-3\frac{2}{15}=$
45. $475\frac{8}{35}-7\frac{9}{28}=$
46. $718\frac{7}{22}-8\frac{6}{23}=$
47. $248\frac{18}{25}-5\frac{2}{3}=$

48. From $585\frac{6}{11}+456\frac{15}{32}+354\frac{17}{30}$ take $8\frac{11}{27}+9\frac{13}{15}+1\frac{1}{11}$.

49. Take $7\frac{7}{8}$ inches from $16\frac{5}{6}$ in., from $18\frac{7}{23}$ in., from $21\frac{5}{17}$ in., from $23\frac{7}{24}$ in., from $29\frac{7}{29}$ in.

50. Take $13\frac{7}{11}$ minutes from $23\frac{1}{6}$, $47\frac{1}{19}$, $31\frac{5}{7}$, $56\frac{17}{23}$, $35\frac{6}{23}$ minutes.

51. Subtract each number from the next to the right; add the first number and all the remainders together. (See No. 1, p. 96.)

$\frac{1}{5}$, $2\frac{1}{2}$, $2\frac{5}{8}$, $3\frac{1}{4}$, $3\frac{3}{8}$, $4\frac{3}{4}$, $5\frac{1}{16}$, $6\frac{1}{4}$, 300.

Applications.—1. A train reached Chicago at 10 o'clock; it had made the trip from Milwaukee in $3^5/_6$ hours. At what time did it start from Milwaukee?

2. A person bought a piece of linen, measuring $60^5/_8$ yards. After the linen was washed it measured only $59^5/_9$ yd. How much had it shrunk?

3. A grocer had 2 cheeses, one weighing $38^7/_8$ and the other $45^{17}/_{32}$ pounds. He sold $7^2/_3$ lb. of each. What was the difference between the weights before and after the sale?

4. What is the difference in age of two persons, now $73^7/_{12}$ and $46^2/_3$ years old respectively? What will it be 10 years hence?

5. A thermometer showed at noon $73^1/_2$ degrees above zero. At 6 o'clock P. M. it showed $65^3/_4$ degrees. How much had it fallen?

6. A grocer received a box of tea weighing $24^7/_{32}$ pounds. The weight of the box alone was $3^9/_{16}$ lb. How much did the tea weigh?

7. From \$$120^3/_5$ the following sums were taken: \$$6^1/_2$, \$$12^2/_5$, \$$26^{17}/_{20}$, \$$20^{11}/_{50}$. What was the remainder?

8. Last fall we received $17^1/_2$ tons of hard coal; in the spring $1^5/_8$ tons were left. How much had we used during the winter?

9. A boy said, "If I had \$$5^2/_3$ more than I have, I should have \$$21^1/_2$." How much did he have?

10. A flag-staff $48^3/_4$ feet high was broken off near the top by a storm, so that it measured only $41^5/_8$ ft. How long was the piece that had been broken off?

11. A farmer, owning $388^2/_3$ acres of land, bought in addition $251^5/_8$, and then sold parcels containing, respectively, $84^3/_4$, $26^7/_8$, $38^4/_5$, $29^{11}/_{12}$, $93^3/_7$, and $84^9/_{10}$ acres. How much had he left?

12. A grocer cut $1^3/_4$, $2^1/_3$, $2^7/_8$, $3^1/_{32}$, $1^7/_8$, $5^{15}/_{16}$, $9^2/_3$, $1^1/_2$, $2^3/_4$, $^1/_{16}$, $^{13}/_{32}$, $2^5/_6$ pounds from a cheese which weighed $43^7/_8$ lb. How many pounds remained?

13. One man cuts $3^2/_3$ cords of firewood per day, another $3^3/_8$ cords per day. The first works 7 days, the second 6 days. How much does one cut more than the other?

Multiplication in Common Fractions.

Example.—1. Three times $^3/_4$ inch are how many inches?

Solution.—$\mathbf{^3/_1\times{}^3/_4}$ in. $=\mathbf{^9/_4}$ in. $=\mathbf{2^1/_4}$ in.

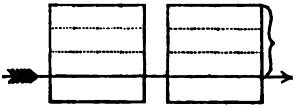
2. Three times $2^3/_4$ inches are how many inches?

Analysis.—$\mathbf{3\times2^3/_4}$ in. $=\mathbf{^3/_1\times{}^{11}/_4}$ in. $=\mathbf{^{33}/_4}$ in. $=\mathbf{8^1/_4}$ in.

3. $^3/_4$ of 2 are how many?

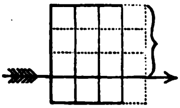
Analysis.—$\mathbf{^3/_4}$ of $\mathbf{^2/_1={}^6/_4=1^1/_2}$.

Note.—After a simple fraction the sign × should be read "of" not "times."



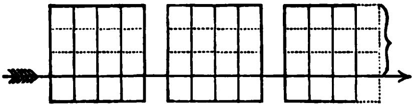
4. $^3/_4$ of $^3/_4$ are how many?

Analysis.—$^1/_4$ of $^1/_4={}^1/_{16}$; $^1/_4$ of $^3/_4={}^3/_{16}$; $\mathbf{^3/_4}$ of $\mathbf{^3/_4={}^9/_{16}}$.



5. $^3/_4$ of $2^3/_4$ are how many?

Illustration.—The line of the arrow cuts off $^1/_4$ of the $2\,^3/_4$ squares represented, leaving $^3/_4$ of the $2\,^3/_4$ squares above it.



Analysis.—$\mathbf{^3/_4}$ of $\mathbf{2^3/_4}={}^3/_4$ of $^{11}/_4$; $^1/_4$ of $^1/_4={}^1/_{16}$; $^1/_4$ of $^{11}/_4={}^{11}/_{16}$; $\mathbf{^3/_4}$ of $\mathbf{^{11}/_4={}^{33}/_{16}=2^1/_{16}}$.

6. $3^3/_4\times2^3/_4$ are how many?

Illustration.—Copy the last diagram four times, omitting the arrow in each line of squares except the fourth. Thus, above the line of the arrow $3\,^3/_4$ times $2\,^3/_4$ squares will be represented, illustrating the following analysis.

Analysis.—$\mathbf{3^3/_4\times2^3/_4}={}^{15}/_4$ of $^{11}/_4$; $^1/_4$ of $^1/_4={}^1/_{16}$; $^1/_4$ of $^{11}/_4={}^{11}/_{16}$; $\mathbf{^{15}/_4}$ of $\mathbf{^{11}/_4={}^{165}/_{16}=10^5/_{16}}$.

The parts of the several analyses printed in **heavy type** are the only parts needed in a written solution. Whence the

147. ***Rule.***—**Reduce integers and mixed numbers to improper fractions. Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.**

For practice in multiplication of fractions, the pupil may complete the following tables, and construct similar ones. When the multiplier is a fraction, he should substitute the word "of" for "times" in all oral exercises.

1	2	3	4	5	6	7
$^1/_2$	1	$1^1/_2$	2	$2^1/_2$	3	$3^1/_2$
$^1/_3$	$^2/_3$	1	$1^1/_3$	$1^2/_3$	2	
$^1/_4$	$^1/_2$	$^3/_4$	1			
$^1/_5$	$^2/_5$	$^3/_5$				
$^1/_6$						
$^1/_7$						

1	$^1/_2$	$^2/_3$	$^3/_4$	$^4/_5$	$^5/_6$	$^6/_7$	$^7/_7$
2	1	$1^1/_3$	$1^1/_2$	$1^3/_5$			
3	$1^1/_2$	2	$2^1/_4$				
4	2	$2^2/_3$	3				
5	$2^1/_2$						
6	3						
7							

1	$^1/_2$	$^1/_3$	$^1/_4$	$^1/_5$	$^1/_6$	$^1/_7$	&c.
$^1/_2$	$^1/_4$	$^1/_6$	$^1/_8$	$^1/_{10}$			
$^1/_3$	$^1/_6$	$^1/_9$					
$^1/_4$	$^1/_8$						
$^1/_5$							
&c.							

1	$^1/_7$	$^2/_7$	$^3/_7$	$^4/_7$	$^5/_7$	$^6/_7$	$^7/_7$
$^1/_5$	$^1/_{35}$	$^2/_{35}$	$^3/_{35}$	$^4/_{35}$			
$^2/_5$	$^2/_{35}$	$^4/_{35}$					
$^3/_5$	$^3/_{35}$				$^3/_7$		
$^4/_5$	$^4/_{25}$						$^4/_5$
$^5/_5$	$^1/_7$	$^2/_7$					1

ORAL EXERCISES.

Note.—Let the oral exercises be carried as far as possible.

1.	2.	3.	4.
$7\times{}^1/_2=$	$^1/_{24}\times 26=$	$6\times{}^2/_3=$	$^7/_{10}\times 23=$
$5\times{}^1/_3=$	$^1/_{61}\times 72=$	$7\times{}^4/_5=$	$^{11}/_{15}\times 60=$
$4\times{}^1/_5=$	$^1/_{25}\times 36=$	$9\times{}^2/_7=$	$^7/_{12}\times 29=$
$6\times{}^1/_4=$	$^1/_{38}\times 47=$	$3\times{}^3/_4=$	$^5/_8\times 13=$
$8\times{}^1/_6=$	$^1/_{27}\times 54=$	$5\times{}^6/_7=$	$^9/_{17}\times 15=$
$9\times{}^1/_7=$	$^1/_{56}\times 63=$	$4\times{}^5/_6=$	$^4/_7\times 25=$
$10\times{}^1/_9=$	$^1/_{21}\times 33=$	$8\times{}^7/_8=$	$^6/_{25}\times 39=$

Note.—In written work, always cancel factors that are common to both numerator and denominator.

5. How much is $6\times{}^1/_{24}$ day? $^1/_6$ d.? $^1/_4$ d.?

6. How much is $9\times{}^3/_8$ lb.? $^3/_4$ lb.? $^{17}/_{16}$ lb.?

7. How much is $17\times\$^1/_4$? $\$^7/_{10}$? $\$^3/_5$?

8. What is $^3/_4$ of 1 hour? 6 h.? 9 h.?

9. What is $^7/_{12}$ of 3, 5, 7 feet?

10.

$^1/_2$ of $^1/_2=$
$^1/_2$ of $^1/_3=$
$^1/_3$ of $^1/_3=$
$^1/_4$ of $^1/_4=$
$^1/_5$ of $^1/_3=$
$^1/_3$ of $^1/_4=$
$^1/_6$ of $^1/_4=$

11.

$^2/_3\times{}^1/_4=$
$^3/_{12}\times{}^1/_2=$
$^3/_4\times{}^1/_5=$
$^5/_{12}\times{}^1/_5=$
$^3/_4\times{}^1/_3=$
$^5/_6\times{}^1/_3=$
$^5/_8\times{}^1/_6=$

12.

$^3/_8$ of $^2/_5=$
$^2/_5$ " $^3/_4=$
$^4/_9$ " $^3/_8=$
$^2/_3$ " $^5/_6=$
$^5/_6$ " $^4/_5=$
$^4/_7$ " $^3/_5=$
$^4/_5$ " $^5/_8=$

13.

$^4/_{21}\times 17=$
$19\times{}^{13}/_{25}=$
$^3/_{11}\times 28=$
$51\times{}^{14}/_{29}=$
$^{27}/_{63}\times 35=$
$46\times{}^{33}/_{34}=$
$^3/_8\times 29=$

14.

$3^1/_8\times5=$
$4^1/_5\times6=$
$5^4/_9\times7=$

15.

$6^5/_{12}\times8=$
$7^7/_8\times9=$
$9^3/_4\times8=$

16.

$^{13}/_3\times{}^{17}/_6=$
$^{19}/_5\times{}^{19}/_8=$
$^{21}/_6\times{}^{11}/_7=$

17.

$^{15}/_2\times{}^{19}/_8=$
$^{29}/_7\times{}^{13}/_9=$
$^{38}/_8\times{}^{31}/_2=$

18. What is $^1/_5$ of $\$^1/_4$? $^1/_5$ of $\$^1/_{10}$? $^1/_5$ of $\$^1/_2$? $^1/_3$ of $\$^1/_8$?

19. What is $^1/_6$ of $^1/_2$ hour? $^1/_6$ of $^1/_5$ h.? $^1/$ of $^1/_6$ h.?

20. What is $^1/_{16}$ of $^1/_2$ lb.? $^1/_{16}$ of $^1/_4$ lb.? $^1/_{16}$ of $^1/_8$ lb.?

21. What is $^2/_3$ of $^1/_4$ ft.? $^3/_4$ of $^1/_2$ ft.? $^5/_8$ of $^1/_3$ ft.?

22. What is $^4/_7$ of $^1/_2$ week? $^1/_4$ of $^3/_7$ wk.? $^3/_5$ of $^5/_6$ wk.?

23. What is $^3/_5$ of $^1/_8$ qt.? $^3/_4$ of $^6/_7$ qt.? $^7/_8$ of $^9/_{10}$ qt.?

24. Multiply $^2/_7$ by $1^1/_2$; $^3/_5$ by $2^3/_4$; $^6/_7$ by $3^3/_5$; $^6/_{11}$ by $4^4/_7$.

25.

$3^1/_2\times{}^1/_4=$
$2^2/_3\times{}^1/_5=$
$4^2/_5\times{}^1/_6=$
$3^3/_4\times{}^1/_8=$

26.

$^1/_7\times7^3/_4=$
$^1/_9\times3^2/_5=$
$^1/_3\times5^1/_2=$
$^1/_5\times4^2/_5=$

27.

$8^2/_3\times{}^5/_6=$
$7^1/_4\times{}^2/_5=$
$54^2/_7\times{}^3/_4=$
$25^3/_8\times{}^5/_6=$

28.

$^3/_8\times6^7/_9=$
$^2/_5\times2^5/_7=$
$^4/_9\times8^3/_5=$
$^7/_{11}\times7^4/_7=$

29. Multiply $3^3/_8$, $7^4/_5$, $9^3/_7$, $6^2/_3$, $8^3/_4$, $5^7/_8$, $4^7/_9$, each by $^{11}/_{23}$.

30. What is $^3/_4$ of $3^{11}/_{12}$? of $5^9/_{11}$? of $6^2/_7$? of $10^2/_3$? of $9^3/_5$?

148. Since mixed numbers may be reduced to improper fractions, and integers may be expressed in fractional form, the general rule may be applied to all cases of multiplication in which fractions are involved, but when the numbers are large, the method is awkward, and requires more figures than the following processes. In business calculations, the rule is seldom followed.

Example.—1. Multiply 85 by $17^2/_3$.

	Analysis.
85	
$17^2/_3$	
3)170	$=2\times 85$
$56^2/_3$	$={}^2/_3\times 85$
595 85	$=17\times 85$
$1501^2/_3$	$=17^2/_3\times 85$

2. Multiply $58^3/_4$ by 29.

	Analysis.
$58^3/_4$	
29	
4)87	$=29\times 3$
$21^3/_4$	$=29\times {}^3/_4$
522 116	$=29\times 58$
$1703^3/_4$	$=29\times 58^3/_4$

3. Multiply $645^2/_3$ by $328^3/_4$.

		Analysis.
	$645^2/_3$	
	$328^3/_4$	
	$^1/_2$	$={}^3/_4\times {}^2/_3$
4)1935=	$483^3/_4$	$={}^3/_4\times 645$
3) 656=	$218^2/_3$	$=328\times {}^2/_3$
	5160 1290 1935	$=328\times 645$
	$212262^{11}/_{12}$	$=328^3/_4\times 645^2/_3$

Explanation.—Beginning at the right, as in multiplication of integers, we multiply separately the fraction and integer of the multiplicand, *first* by the fraction and *second* by the integer of the multiplier.

The work at the left indicates the steps by which we obtain the product of the integers and fractions.

ORAL EXERCISES.

Multiply

1. $8^1/_3$ by 12
$15^1/_4$ by 16
$7^2/_3$ by 9
$6^1/_2$ by 14

2. $13^1/_3$ by 9
$17^2/_3$ by 6
$18^3/_4$ by 8
$15^3/_4$ by 12

3. 6 by $2^1/_3$
12 by $3^2/_3$
14 by $6^5/_7$
16 by $4^5/_8$

4. 12 by $7^2/_3$
18 by $4^5/_6$
21 by $6^5/_7$
15 by $7^3/_5$

5. $2^1/_2$ by 3
$4^2/_3$ by 5
$7^1/_5$ by 6

6. $4^1/_6$ by 5
$6^5/_7$ by 8
$8^5/_{12}$ by 7

7. 9 by $3^1/_2$
13 by $6^2/_5$
18 by $3^1/_8$

8. 37 by $2^1/_4$
16 by $6^3/_7$
27 by $2^3/_8$

SLATE EXERCISES.

1.	2.	3.	4.
$3^{1}/_{5}\times 6^{2}/_{3}=$	$6^{1}/_{5}\times 4^{2}/_{5}=$	$14^{2}/_{3}\times 11^{3}/_{10}=$	$1^{17}/_{25}\times 3^{10}/_{23}=$
$2^{3}/_{4}\times 3^{1}/_{2}=$	$44^{2}/_{5}\times 5^{1}/_{6}=$	$13^{1}/_{8}\times 25^{7}/_{9}=$	$7^{13}/_{16}\times 6^{19}/_{27}=$
$5^{2}/_{3}\times 2^{2}/_{5}=$	$82^{3}/_{8}\times 7^{1}/_{2}=$	$16^{2}/_{5}\times 13^{3}/_{7}=$	$6^{23}/_{27}\times 9^{15}/_{16}=$
$7^{1}/_{2}\times 3^{3}/_{4}=$	$27^{6}/_{7}\times 3^{3}/_{8}=$	$24^{3}/_{8}\times 28^{5}/_{9}=$	$9^{18}/_{19}\times 7^{28}/_{29}=$

5. Find the product of $2^{2}/_{5}\times \$30^{7}/_{10}$; $1^{4}/_{5}\times \$25^{5}/_{9}$; $4^{7}/_{8}\times \$19^{2}/_{3}$.

6. Multiply $\$18^{2}/_{3}$ by $15^{3}/_{7}$; $\$27^{13}/_{21}$ by $38^{17}/_{20}$; $41^{1}/_{2}$ by $20^{1}/_{2}$.

7. $2^{1}/_{25}\times 5^{3}/_{4}\times 16^{2}/_{3}=?$ $^{3}/_{4}$ of $^{2}/_{5}$ of $^{5}/_{6}$ of $^{8}/_{21}$ of $^{9}/_{5}=?$

8. What number is $3^{7}/_{9}$ times $35^{1}/_{2}$? $5^{4}/_{7}$ times $6^{3}/_{8}$?

9. There are $4^{1}/_{2}$ lb. in a package. How many pounds in $8^{1}/_{4}$ such packages?

10. Reduce $^{2}/_{3}\times{}^{3}/_{4}\times{}^{4}/_{5}\times{}^{5}/_{9}\times{}^{9}/_{32}\times{}^{16}/_{17}$ to a simple expression.

Applications.—Note.—In business it is customary to drop a fraction in the result, if less than $^{1}/_{2}$¢, and to add 1¢ to the integer, if the fraction is equal to or greater than $^{1}/_{2}$¢. The pupil should here be required to obtain the exact answer, and to write the result in business form under it.

11–38. *What is the cost of*

1 $37^{1}/_{2}$ bushels of potatoes at $75^{1}/_{2}$¢?
2 345 yards of cloth at $90^{3}/_{4}$¢?
3 $17^{1}/_{2}$ feet of oilcloth at $3^{1}/_{5}$¢?
4 $43^{1}/_{2}$ quarts of cider at $5^{1}/_{6}$¢?
5 387 bushels of oats at $43^{3}/_{4}$¢?
6 $40^{1}/_{16}$ pounds of starch at $18^{1}/_{2}$¢?
7 345 barrels of apples at $\$3^{2}/_{5}$?
18 $47^{1}/_{2}$ sacks of flour at $\$2^{3}/_{5}$?
19 $53^{1}/_{2}$ pounds of cheese at $9^{3}/_{4}$¢?
20 17 sacks of rice at $\$14^{3}/_{5}$?
64 pecks of beans at $17^{3}/_{5}$¢?
$27^{4}/_{5}$ pecks of potatoes at $23^{1}/_{2}$¢?
$328^{3}/_{4}$ pounds of butter at $43^{3}/_{4}$¢?
$17^{9}/_{16}$ pounds of bacon at $12^{1}/_{2}$¢?
15 $56^{1}/_{2}$ pounds of tea at $\$^{3}/_{4}$?
$6^{3}/_{4}$ tons of coal at $\$3^{3}/_{4}$?
$15^{1}/_{2}$ yards of ribbon at $37^{1}/_{2}$¢?
$24^{1}/_{4}$ gallons of oil at 85¢?
3 quarts of nuts at $12^{1}/_{2}$¢
300 bushels of rye at $94^{1}/_{2}$¢?
$12^{1}/_{2}$ yards of lace at $\$5^{1}/_{4}$?
$17^{3}/_{4}$ pounds of honey at $18^{3}/_{4}$¢?
$4^{1}/_{2}$ barrels of herring at $\$3^{1}/_{4}$?
325 pounds of beef at $11^{1}/_{2}$¢?
63 bl. of cranberries at $\$12^{1}/_{2}$?
$17^{3}/_{8}$ bu. of strawberries at $\$4^{5}/_{8}$?
$37^{5}/_{8}$ yards of velvet at $\$4^{3}/_{8}$?
$48^{3}/_{4}$ yards of carpet at $\$1^{3}/_{4}$?

39. How many square feet in a square, each side of which measures $3^{7}/_{8}$ feet? $3^{7}/_{8}$ inches? $3^{7}/_{8}$ yards?

Draw a square, each side measuring $2^{3}/_{4}$ inches. Divide each side into fourths of an inch, and draw lines, cutting the square into small ones, each a fourth of an inch square. How many of these small squares? How many square inches?

40. Find the number of square inches in a rectangle $5^{7}/_{8}$ ft. long and $7^{3}/_{4}$ inches wide.

41. What is the cost of 9 tons of coal at $\$3^{3}/_{5}$, with cartage at $\$^{1}/_{4}$ per ton?

42. If the salary of an officer is $1700, how much does he receive in $^{1}/_{4}$, $^{1}/_{6}$, $^{1}/_{10}$, $^{1}/_{12}$ year? How much in $^{2}/_{3}$, $^{3}/_{4}$, $^{4}/_{5}$, $^{7}/_{12}$ year?

43. A laborer earns $\$20^{3}/_{5}$ a week. How much in $^{3}/_{5}$ year? In $1^{4}/_{5}$ yr.? (Count 52 weeks.)

44. My neighbor pays $700 rent per year. How much is that per month?

45. I buy some lots containing respectively $^{3}/_{4}$, $^{4}/_{5}$, $^{13}/_{25}$, $^{47}/_{50}$, $6^{3}/_{10}$, $19^{13}/_{20}$ acres. What is the cost of the whole, at $48 per acre?

46. How many square feet in the surface of a stone slab $2^{7}/_{8}$ feet wide and $4^{5}/_{6}$ feet long?

47. Four boxes of hardware, weighing respectively $3^{1}/_{4}$ cwt., $4^{3}/_{5}$ cwt., $4^{1}/_{2}$ cwt., and $4^{3}/_{10}$ cwt., cost 16¢ freight per cwt. What is the freight on each box, and on the 4 boxes?

Find the sum to be paid for

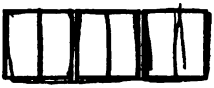
48. $5^{1}/_{2}$ lb. sugar, at $9^{1}/_{2}$¢
$6^{3}/_{4}$ lb. coffee, at $32^{1}/_{2}$¢
$2^{4}/_{5}$ lb. rice, at $8^{4}/_{7}$¢
$14^{3}/_{8}$ lb. flour, at $4^{1}/_{5}$¢
$3^{3}/_{4}$ lb. butter, at $23^{1}/_{2}$¢
$2^{1}/_{2}$ lb. cheese, at $11^{3}/_{4}$¢
$3^{5}/_{6}$ doz. eggs, at 20¢
$2^{4}/_{5}$ lb. starch, at $12^{1}/_{2}$¢

49. 18 yd. calico, at 9¢
$20^{1}/_{2}$ yd. alpaca, at $42^{1}/_{2}$¢
$19^{1}/_{4}$ yd. shirting, at 17¢
25 yd. ribbon, at $33^{1}/_{2}$¢
10 doz. buttons, at $\$^{1}/_{4}$
3 cloaks, at $\$18^{2}/_{5}$
10 yd. velvet, at $\$3^{3}/_{4}$
$22^{1}/_{2}$ yd. velveteen, at $\$1^{4}/_{5}$

11
95
19
385

Division in Fractions.

Note.—In the first two exercises the square is the unit. In the third exercise the linear inch is the unit.



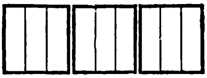
1. *a.* How many times 1 in 3, $2\frac{1}{2}$, 2, $1\frac{1}{2}$? What part of 1 is contained in $\frac{1}{2}$? What part of 2 in $\frac{1}{2}$? In $1\frac{1}{2}$? etc.

b. How many times $\frac{1}{2}$ in 1? In 2? In 3? How many times $1\frac{1}{2}$ in 3? $2\frac{1}{2}$ in 3? ($\frac{5}{2}$ in $\frac{6}{2}$?)

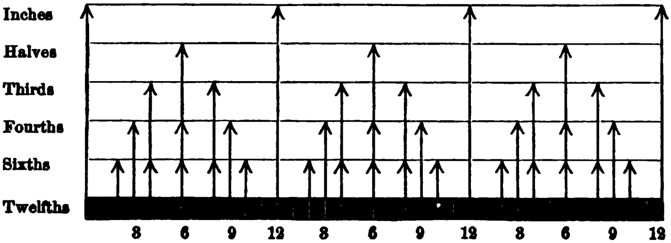
c. How many times $\frac{1}{2}$ in $1\frac{1}{2}$? In $2\frac{1}{2}$? $1\frac{1}{2}$ in $2\frac{1}{2}$?

2. *a.* How many times 1 in $1\frac{2}{3}$? In $2\frac{1}{3}$? What part of 1 is in $\frac{2}{3}$? What part of 2? What part of 2 is in $\frac{1}{3}$? In $1\frac{1}{3}$?

b. How many times $\frac{1}{3}$ in 1? In 2? How many times $\frac{2}{3}$ in 2? In 3? How many times $1\frac{2}{3}$ in 3? $2\frac{1}{3}$ in 3?



c. How many times $\frac{1}{3}$ in $\frac{5}{3}$? In $2\frac{2}{3}$? How many times $1\frac{1}{3}$ in $2\frac{2}{3}$? $\frac{2}{3}$ in $2\frac{2}{3}$? $1\frac{2}{3}$ in 3? $1\frac{1}{3}$ in $2\frac{1}{3}$?

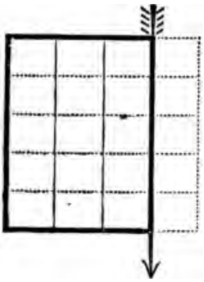


Note.—The points of arrows divide the inches into halves, thirds, etc. By following the shafts downward equivalents in smaller parts are found.

3. *a.* How many times 2 in $2\frac{3}{4}$? What part of 3 in $\frac{1}{2}$, $1\frac{2}{3}$?

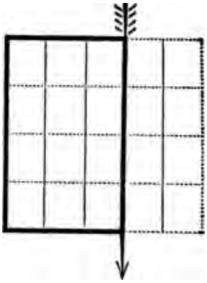
b. How many times $\frac{1}{6}$ in 2? In $2\frac{1}{2}$? In 3? How many times $\frac{3}{4}$ in 1? In $\frac{7}{12}$? In 2? How many times $\frac{5}{6}$ in 3? What part of 2 is $\frac{1}{4}$, $\frac{2}{3}$? What part of 3 is $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$?

c. How many times $\frac{1}{4}$ in $\frac{1}{2}$? $\frac{5}{12}$ in $\frac{2}{3}$? $\frac{1}{3}$ in $\frac{3}{4}$? $\frac{1}{4}$ in $\frac{2}{3}$? What part of $\frac{1}{2}$ is in $\frac{1}{4}$, $\frac{1}{3}$? What part of $\frac{2}{3}$ in $\frac{3}{4}$? etc.



4. Divide $\frac{3}{4}$ by $\frac{3}{5}$.

Illustration.—To find how many times $\frac{3}{5}$ is contained in $\frac{3}{4}$, the 5ths and 4ths are subdivided into parts of the same size by dividing each 5th into four and each 4th into five equal parts, thus making $\frac{12}{20}$ and $\frac{15}{20}$, whence we see that $\frac{3}{5}$ is contained in $\frac{3}{4}$ as many times as 12 is contained in $15 = \frac{15}{12} = 1\frac{1}{4}$.



149. The arithmetical solution is performed on the same principle as the solution by diagram, the folding of paper, etc., thus:

Indicating the division by writing the divisor under the dividend (see Art. 72, § 3), we reduce both fractions to 20ths, 20 being the least common multiple of 4 and 5, and divide the numerator of the dividend by the numerator of the divisor.

$$\frac{\dfrac{3\times5}{\cancel{4\times5}}=\dfrac{15}{\cancel{20}}}{\dfrac{3\times4}{\cancel{5\times4}}=\dfrac{12}{\cancel{20}}}=\frac{15}{12}=1\frac{1}{4}$$

But, since the common denominator does not affect the quotient ($\frac{15}{12}$), the work, printed in italics, by which it is obtained may be omitted. The written process would then appear as at the right.

$$\frac{3}{4}\times\frac{5}{3}=\frac{15}{12}=1\frac{1}{4}$$

150. ***Rule.***—**Invert the divisor and multiply the numerators together for the numerator of the quotient, and the denominators for the denominator of the quotient.**

Note.—Since integers and mixed numbers may be expressed in the form of improper fractions, this rule applies to all cases of Division in Common Fractions.

151. The following analysis leads to an equivalent rule:

$\frac{3}{5}$ is contained in 1, or $\frac{5}{5}$, five thirds times, and in $\frac{3}{4}$ it is contained $\frac{3}{4}$ of $\frac{5}{3}$ times; hence we have

$$\frac{3}{4}\div\frac{3}{5}=\frac{3}{4}\times\frac{5}{3}=\frac{15}{12}=1\frac{1}{4}.$$

152. An inverted fraction shows how many times the fraction is contained in one, and is called the ***reciprocal*** of the fraction. Hence, for dividing one fraction by another, we have the

Rule.—**Multiply the reciprocal of the divisor by the dividend.**

153. In dividing a fraction by an integer, a part of the written work required by the rule may be omitted, as follows :

First, when the numerator of the fraction is divisible by the integer, as in

Example.—**1.** Divide $^6/_7$ by 3. (What part of 3 in $^6/_7$?)

The process of division, according to the rule, would be

$$6/_7 \div 3 = 1/_{\cancel{3}} \times \overset{2}{\cancel{6}}/_7 = 2/_7,$$

but this is equivalent to dividing the numerator directly by the integer, thus,

$$6/_7 \div 3 = 2/_7.$$

Second, when the numerator of the fraction is not divisible by the integer, as in

Example.—**2.** Divide $^5/_7$ by 3. (What part of 3 in $^5/_7$?)

The process of division, under the rule, would be

$$5/_7 \div 3 = 1/_3 \times 5/_7 = 5/_{21},$$

but we can multiply the denominator directly by 3 without writing out the second step; hence, we need to put down only

$$5/_7 \div 3 = 5/_{21}.$$

154. *Hence, dividing the numerator or multiplying the denominator of a fraction divides the value of the fraction.*

155. In dividing a mixed number by an integer, by a fraction, or by a mixed number, the written work may be as follows :

Example.—**3.** Divide $379^3/_4$ by 6.

$$\begin{array}{r} 6)\underline{379^3/_4} \\ 63^7/_{24} \end{array}$$

Explanation.—Six is contained in $379^3/_4$ 63 times, with a remainder of $1^3/_4$.

$1^3/_4 = {^7/_4}$; $^7/_4 \div 6 = {^7/_{24}}$, which, being annexed to 63, gives the entire quotient, $63^7/_{24}$.

Note.—In the two following examples we multiply both divisor and dividend by the denominator of the divisor in order to get rid of the fraction in the divisor. This makes the division more convenient and does not alter the value of the quotient. The process then becomes the same as in the preceding solution.

4. Divide $379^1/_4$ by $^2/_3$.

$$\begin{array}{l} ^2/_3)\ 379^1/_4 \\ \underline{3 \quad\ 3} \\ 2)\underline{1137^3/_4} \\ \quad\ 568^7/_8 \end{array}$$

11

5. Divide $349^2/_3$ by $2^1/_4$.

$$\begin{array}{l} 2^1/_4)349^2/_3 \\ \underline{4 \qquad 4} \\ 9)\ \underline{1398^2/_3} \\ \qquad 155^{11}/_{27} \end{array}$$

ORAL EXERCISES.

Divide

1.	2.	3.	4.
$^1/_4$ by $^1/_{36}$	$^1/_6$ by $^1/_{12}$	$^2/_3$ by $^1/_{18}$	$^1/_2$ by $^1/_{36}$
$^1/_4$ by $^1/_{28}$	$^1/_6$ by $^1/_{18}$	$^2/_3$ by $^1/_{15}$	$^1/_2$ by $^1/_{34}$
$^1/_4$ by $^1/_{24}$	$^1/_6$ by $^1/_{24}$	$^2/_3$ by $^1/_{12}$	$^1/_2$ by $^1/_{32}$
$^1/_4$ by $^1/_{16}$	$^1/_6$ by $^1/_{36}$	$^2/_3$ by $^1/_9$	$^1/_2$ by $^1/_{30}$

Question.—In which of the above columns do the quotients increase as you descend? Why do they increase? In which do they decrease? Why?

5. Divide $^3/_4$ by $^1/_6$; $^2/_3$ by $^1/_5$; $^3/_5$ by $^1/_4$; $^5/_6$ by $^1/_4$; $^6/_{11}$ by $^5/_{11}$.

6. " $^3/_4$ by $^1/_7$; $^5/_8$ by $^5/_9$; $^3/_5$ by $^8/_{11}$; $^2/_{47}$ by $^5/_{47}$; $^7/_8$ by $^1/_9$.

7. " $^2/_3$ by $^9/_{32}$; $^5/_8$ by $^7/_{12}$; $^9/_{20}$ by $^1/_4$; $^7/_{23}$ by $^2/_6$; $^1/_4$ by $^1/_{27}$.

8. How many times may $^1/_3$ of a pound be taken from $^6/_7$ lb., $^3/_5$ lb., $^7/_8$ lb., $^9/_{10}$ lb.?

9.	10.	11.	12.
$^3/_4 \div 4 =$	$^4/_{19} \div 2 =$	$^7/_8 \div 4 =$	$^{11}/_{15} \div 7 =$
$^1/_2 \div 3 =$	$^8/_{13} \div 9 =$	$^{65}/_{66} \div 13 =$	$^{19}/_{27} \div 3 =$
$^5/_6 \div 7 =$	$^5/_9 \div 2 =$	$^{63}/_{71} \div 7 =$	$^1/_2 \div 4 =$
$^9/_{11} \div 3 =$	$^3/_4 \div 3 =$	$^{64}/_{75} \div 16 =$	$^3/_4 \div 9 =$
$^5/_9 \div 6 =$	$^{12}/_{13} \div 4 =$	$^6/_7 \div 8 =$	$^4/_5 \div 8 =$

13. Divide $6^2/_3$ by $4^1/_8$; $5^1/_5$ by $2^7/_9$; $3^8/_{11}$ by $1^1/_{13}$; $2^1/_2$ by $^1/_4$.

14. Divide $9^9/_{10}$ by $4^1/_7$; $2^3/_5$ by $9^9/_{10}$; $7^7/_8$ by $2^4/_5$; $6^7/_9$ by $3^3/_4$.

15.	16.	17.	18.
$2^3/_4 \div 7 =$	$5^6/_7 \div 9 =$	$9^4/_5 \div 8 =$	$8^1/_3 \div 5 =$
$6^1/_5 \div 8 =$	$3^1/_3 \div 7 =$	$8^1/_7 \div 8 =$	$3^4/_7 \div 5 =$
$9^2/_3 \div 5 =$	$7^2/_5 \div 6 =$	$7^5/_8 \div 6 =$	$4^5/_6 \div 8 =$

19. Divide 5 by $^3/_4$; 7 by $^9/_{11}$; 6 by $^5/_6$; 12 by $^3/_4$.

20.	21.	22.	23.
$18 \div {^7/_8} =$	$14 \div {^7/_8} =$	$39 \div {^6/_{11}} =$	$27 \div {^7/_{13}} =$
$36 \div {^7/_9} =$	$41 \div {^7/_{19}} =$	$23 \div {^5/_{13}} =$	$20 \div {^7/_{10}} =$
$84 \div {^2/_3} =$	$35 \div {^7/_9} =$	$49 \div {^4/_7} =$	$19 \div {^7/_{15}} =$
$17 \div {^5/_6} =$	$71 \div {^3/_5} =$	$21 \div {^5/_8} =$	$23 \div {^8/_{17}} =$

24. Divide 23 by $4^1/_2$; 17 by $5^2/_3$; 48 by $3^7/_8$; 59 by $7^1/_5$.

25. Divide 25 by $3^1/_3$; 24 by $7^1/_6$; 39 by $8^2/_3$; 72 by $2^5/_6$.

ORAL AND SLATE EXERCISES.

Note.—Let the oral exercises be carried as far as time and the ability of the pupils will permit.

1. $1 \div 6 =$
$1 \div 8 =$
$2 \div 3 =$
$2 \div 4 =$
$5 \div 7 =$

2. $4 \div 3 =$
$7 + 4 =$
$6 \div 5 =$
$14 \div 8 =$
$8 \div 5 =$

3. $4 \div \frac{1}{2} =$
$3 \div \frac{1}{4} =$
$2 \div \frac{1}{6} =$
$5 \div \frac{1}{9} =$
$7 \div \frac{1}{8} =$

4. $5 \div \frac{2}{3} =$
$6 \div \frac{3}{7} =$
$8 \div \frac{5}{8} =$
$4 \div \frac{4}{5} =$
$10 \div \frac{2}{9} =$

5. $3 \div 1\frac{1}{2} =$
$5 \div 2\frac{2}{3} =$
$27 \div 7\frac{3}{4} =$
$15 \div 3\frac{2}{5} =$
$39 \div 9\frac{3}{5} =$

6. $\frac{2}{3} \div 4 =$
$\frac{3}{4} \div 5 =$
$\frac{4}{6} \div 3 =$
$\frac{3}{7} \div 6 =$

7. $\frac{7}{8} \div \frac{1}{8} =$
$\frac{8}{9} \div \frac{2}{9} =$
$\frac{6}{7} \div \frac{3}{7} =$
$\frac{14}{15} \div \frac{7}{15} =$

8. $2\frac{2}{5} \div \frac{3}{5} =$
$3\frac{3}{4} \div \frac{3}{4} =$
$3\frac{9}{11} \div \frac{7}{11} =$
$5\frac{5}{9} \div \frac{5}{9} =$

9. $15\frac{1}{2} \div 4\frac{1}{2} =$
$17\frac{4}{5} \div 3\frac{2}{5} =$
$16\frac{2}{3} \div 5\frac{2}{3} =$
$29\frac{4}{7} \div 9\frac{3}{7} =$

10. How many times $\$4\frac{3}{4}$ in $\$26\frac{1}{4}$? In $\$37\frac{3}{4}$? In $\$45\frac{2}{4}$?

11. $3\frac{3}{7}$ hours is what part of $19\frac{1}{7}$, $27\frac{2}{7}$, $38\frac{3}{7}$ hours?

12. What part of $18\frac{9}{16}$, $24\frac{12}{16}$, $30\frac{15}{16}$, $49\frac{8}{16}$ lb. is $6\frac{3}{16}$ lb.?

13. $\frac{1}{2} \div \frac{1}{4} =$
$\frac{1}{4} \div \frac{1}{8} =$
$\frac{1}{9} \div \frac{1}{36} =$
$\frac{1}{4} \div \frac{1}{20} =$
$\frac{1}{3} \div \frac{1}{9} =$

14. $\frac{10}{21} \div \frac{2}{3} =$
$\frac{12}{24} \div \frac{4}{6} =$
$\frac{20}{35} \div \frac{5}{7} =$
$\frac{35}{42} \div \frac{5}{6} =$
$\frac{18}{33} \div \frac{6}{11} =$

15. $\frac{5}{6} \div \frac{1}{18} =$
$\frac{4}{5} \div \frac{1}{15} =$
$\frac{5}{6} \div \frac{1}{9} =$
$\frac{4}{5} \div \frac{1}{25} =$
$\frac{5}{7} \div \frac{1}{28} =$

16. $\frac{4}{5} \div \frac{2}{25} =$
$\frac{8}{21} \div \frac{4}{63} =$
$\frac{25}{36} \div \frac{5}{72} =$
$\frac{4}{5} \div \frac{5}{25} =$
$\frac{7}{18} \div \frac{7}{54} =$

17. $\frac{1}{6} \div \frac{1}{12}$; $\frac{1}{72} \div \frac{1}{18}$; $\frac{1}{75} \div \frac{1}{15}$; $\frac{1}{64} \div \frac{1}{16}$; $\frac{5}{36} \div \frac{5}{9}$.

18. $\frac{1}{3} \div \frac{1}{5}$; $\frac{1}{2} \div \frac{1}{6}$; $\frac{1}{4} \div \frac{1}{15}$; $\frac{1}{18} \div \frac{1}{9}$; $\frac{1}{3} \div \frac{3}{4}$.

19. $\frac{1}{9} \div \frac{2}{7}$; $\frac{3}{8} \div \frac{5}{6}$; $\frac{3}{5} \div \frac{2}{5}$; $\frac{2}{7} \div \frac{4}{9}$; $\frac{3}{7} \div \frac{5}{12}$.

20. $1\frac{1}{3} \div \frac{2}{9} =$
$3\frac{3}{4} \div \frac{5}{12} =$
$2\frac{1}{2} \div \frac{5}{16} =$
$4\frac{4}{5} \div \frac{4}{15} =$
$1\frac{3}{4} \div \frac{7}{20} =$

21. $11\frac{1}{3} \div 1\frac{2}{15} =$
$23\frac{1}{2} \div 2\frac{1}{8} =$
$45\frac{2}{5} \div 3\frac{2}{25} =$
$25\frac{1}{2} \div 4\frac{1}{4} =$
$27\frac{1}{3} \div 2\frac{1}{6} =$

22. $3\frac{2}{3} \div \frac{5}{7} =$
$2\frac{1}{14} \div \frac{4}{9} =$
$1\frac{1}{6} \div \frac{3}{5} =$
$1\frac{4}{5} \div \frac{7}{13} =$
$2\frac{7}{15} \div \frac{4}{51} =$

23. $5\frac{1}{12} \div 4\frac{1}{5} =$
$3\frac{1}{4} \div 2\frac{1}{8} =$
$5\frac{1}{10} \div 1\frac{4}{5} =$
$6\frac{2}{3} \div 3\frac{2}{15} =$
$7\frac{1}{3} \div 4\frac{1}{16} =$

24. Divide $\frac{4}{7}$ by $2\frac{2}{3}$; $\frac{1}{8}$ by $3\frac{3}{4}$; $\frac{2}{3}$ by $1\frac{4}{9}$; $\frac{1}{9}$ by $2\frac{2}{5}$; $\frac{1}{5}$ by $3\frac{3}{4}$.

25. Divide $\frac{3}{4}$ by $7\frac{4}{5}$; $\frac{4}{9}$ by $3\frac{3}{4}$; $\frac{5}{7}$ by $4\frac{4}{5}$; $\frac{5}{8}$ by $5\frac{5}{6}$; $\frac{3}{8}$ by $3\frac{2}{5}$.

Applications.—1. If 4 yards of ribbon cost $\$^5/_7$, what will 1 yard cost?

Analysis.—If 4 yards cost $\$^5/_7$, one yard will cost $^1/_4$ of $\$^5/_7 = \$^5/_{28}$.

2. A farmer sold 5 dozen eggs for $\$^{11}/_{20}$. How much was that per dozen?

3. In six days a man plows $^5/_{14}$ of a field. At this rate, how much does he plow in 1 day?

4. If a weaver earns $\$9^3/_{20}$ per week (6 days), what does she earn per day?

5. If $^4/_5$ lb. of coffee cost $\$^3/_8$, what will 1 lb. cost?

Analysis.—If $^4/_5$ lb. cost $\$^3/_8$, $^1/_5$ will cost $^1/_4 \times \$^3/_8 = \$^3/_{32}$, and $^5/_5$ lb. will cost $5 \times {}^3/_{32} = \$^{15}/_{32}$.

6. If a traveler can make $^5/_6$ of his journey in $^3/_7$ of a month, what time will the entire journey require?

7. If $^2/_9$ of a bar of gold weighs $^5/_{12}$ lb., what is the weight of the bar?

8. How many are $^2/_3$ of $2^1/_2$ dozen? $^2/_3$ of $2^1/_2$ gross?

9. A garden, containing $148^4/_5$ □ yd., is to be divided up into beds of $12^2/_5$ □ yd. each. How many such beds will there be?

10. A quantity of grain, weighing $110^1/_4$ cwt., is to be put into bags, each holding $1^3/_4$ cwt. How many bags are required?

11. How many yards of cloth at $\$^3/_8$ a yard can be bought for $2, $5, $7, $9, $4, $23?

12. How many bushels of potatoes at $\$^{24}/_{25}$ per bu. can be bought for $6, $8, $11; $13?

13. How many times may $1^8/_9$ quarts be drawn from a can holding 17 quarts? From one holding $22^2/_3$ quarts?

14. If a laborer can mow a field in $7^{14}/_{19}$ days, how much of it can he mow in 1 day?

15. Divide $22,500 among the 7 members of a family so that each of the 4 older ones may receive a third more than one of the younger.

Complex Fractions.

If a slip of paper, a melon, or other object, is cut into 3 equal parts, one and a half of these parts are $1\frac{1}{2}$ thirds, $2\frac{1}{2}$ parts are $2\frac{1}{2}$ thirds, etc.

Show by the use of objects what is meant by $\frac{2\frac{1}{3}}{5}$, $\frac{1\frac{3}{4}}{3}$, etc.

156. ***A complex fraction*** is a fraction having a fraction or mixed number for its numerator and an integer for its denominator.

Reducing Complex to Simple Fractions. **Example.**—1. Reduce $\frac{2\frac{1}{3}}{4}$ to a simple fraction.

Analysis.—If each fourth of an object be divided into 3 equal parts, 4 fourths, or the whole, will contain 4 times 3, or 12, and $2\frac{1}{3}$ will contain $2\frac{1}{3}$ times 3, or 7, of them, hence $2\frac{1}{3}$ fourths are equal to $\frac{7}{12}$.

Written Work.

$$\frac{2\frac{1}{3}}{4}=\frac{2\frac{1}{3}\times3}{4\times3}=\frac{7}{12}$$

Illustration.—The mode of demonstrating the foregoing analysis by means of objects is sufficiently indicated by the analysis.

Reduce to simple fractions:

1. $\frac{2\frac{1}{3}}{6}$ 2. $\frac{3\frac{1}{2}}{8}$ 3. $\frac{1\frac{2}{3}}{7}$ 4. $\frac{5\frac{1}{2}}{9}$ 5. $\frac{2\frac{3}{4}}{3}$ 6. $\frac{3\frac{3}{4}}{2}$

7. $\frac{\frac{2}{5}}{7}$ 8. $\frac{4\frac{1}{7}}{8}$ 9. $\frac{6\frac{1}{3}}{11}$ 10. $\frac{\frac{3}{9}}{15}$ 11. $\frac{\frac{1}{8}}{12}$ 12. $\frac{7\frac{3}{8}}{10}$

13. $\frac{2356\frac{7}{9}}{3872}$ 14. $\frac{483\frac{1}{13}}{286}$ 15. $\frac{\frac{3}{5}}{989}$

157. Expressions in which fractions or mixed numbers occur as denominators, as $\frac{4}{5\frac{1}{2}}$, are not properly fractions, though they are commonly classified as such. They only indicate division (see Art. 72), and are reduced by performing the division indicated, or by multiplying divisor and dividend of the complex expression by the least common multiple of the denominators of their fractional parts (see Art. 85).

SLATE EXERCISES.

Reduce to simple fractions:

1. $\dfrac{2\frac{1}{3}}{3\frac{1}{4}}$ 2. $\dfrac{5\frac{1}{2}}{7\frac{3}{5}}$ 3. $\dfrac{6\frac{2}{3}}{3\frac{1}{3}}$ 4. $\dfrac{33\frac{1}{8}}{51\frac{2}{11}}$ 5. $\dfrac{68\frac{1}{5}}{97\frac{2}{3}}$

6. $\dfrac{17\frac{3}{7}}{18\frac{1}{7}}$ 7. $\dfrac{32\frac{3}{4}}{33\frac{3}{8}}$ 8. $\dfrac{6\frac{5}{12}}{9\frac{3}{4}}$ 9. $\dfrac{13\frac{2}{3}}{6\ldots}$ [illegible] 10. $\dfrac{17\frac{1}{7}}{23\frac{2}{3}}$

The reduction of the following complicated expressions will afford exercises in addition, subtraction, multiplication, and division of fractions.

1. $\frac{3}{5}+(2\frac{1}{3} \text{ of } 2\frac{1}{5})+(\frac{4}{5}\times 2\frac{1}{3})+(6\frac{3}{13}\div\frac{45}{91})$

2. $4\frac{2}{9}\times 6\frac{3}{7}\div\dfrac{2\frac{1}{2}}{7}$ 3. $\dfrac{\frac{5}{7}+\frac{1}{4}}{2\frac{1}{2}-1\frac{6}{7}}$ 4. $\dfrac{\frac{14}{27}\times\frac{18}{35}}{1\frac{23}{25} \text{ of } \frac{5}{36}}$

5. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$ 6. $6\frac{2}{3}\div\dfrac{3-\frac{16}{31}}{\frac{7}{8}-\frac{3}{5}}$ 7. $\dfrac{1}{6+1\frac{7}{8}}$

8. $\dfrac{1\frac{4}{17}\times 6\frac{4}{5}}{3\frac{3}{5}-1\frac{2}{25}}+(\frac{2}{11} \text{ of } 2\frac{2}{5})-\frac{3}{4}-\dfrac{1\frac{3}{5}}{12}$ 9. $3\frac{1}{3}\div\dfrac{1+\frac{15}{16}}{\frac{1}{5}+\frac{1}{8}}$

10. $\dfrac{\frac{1}{3}+\frac{4}{9}+\frac{11}{27}}{\frac{1}{4}+\frac{5}{6}+\frac{11}{12}} \text{ of } \dfrac{\frac{5}{8} \text{ of } 2\frac{1}{3}\div 1\frac{11}{24}}{\frac{16}{27} \text{ of } 4\frac{5}{8}\div 9\frac{1}{4}}$ 11. $\dfrac{15\frac{3}{4}-(4\frac{2}{3}\times 1\frac{5}{6})}{(\frac{1}{5}\div\frac{3}{70})+2\frac{19}{36}}$

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158. Applications.—To find what part one given number is of another.

Example.—1. What part of 8 is 3?

Analysis.—3 is $\frac{3}{8}$ of 8 because it is 3 of the 8 equal parts into which 8 can be divided. Illustrate by the use of counters.

What part of

2. 7 is 5? 16 is 12? 18 is 15? 21 is 14? 30 is 20?

3. 39 is 26? 42 is 28? 48 is 36? 72 is 48? 32 is 24?

Example.—4. What part of 5 is $\frac{2}{3}$?

Analysis.—$5 = \frac{15}{3}$, and $\frac{2}{3}$ is $\frac{2}{15}$ of $\frac{15}{3}$ because it is 2 of the 15 equal parts into which 15 thirds can be divided.

Note.—Illustrate with objects. By cutting 5 wholes into thirds, and taking 2 *of the fifteen*, we see how nearly this problem is like the preceding.

What part of

5. 4 is $^3/_4$? 12 is $^2/_3$? 15 is $^5/_9$? 18 is $^2/_3$? 25 is $^5/_8$?

6. 39 is $^3/_5$? 16 is $^4/_5$? 21 is $^3/_7$? 28 is $^4/_9$? 36 is $^{12}/_{13}$?

Example.—7. What part of $^4/_5$ is $^2/_3$?

Analysis.—$^4/_5 = {}^{12}/_{15}$ and $^2/_3 = {}^{10}/_{15}$, $^{10}/_{15}$ is $^{10}/_{12}$ of $^{12}/_{15}$ because it is 10 of the 12 equ[illegible]rts into which 12 fifteenths can be divided. Illustrate with objects.

What part of

8. $^5/_6$ is $^3/_8$? $^6/_7$ is $^2/_3$? $^3/_5$ is $^5/_9$? $^3/_4$ is $^4/_7$? $^5/_{11}$ is $^1/_5$?

9. $3^2/_3$ is $1^1/_2$? $2^1/_3$ is $1^1/_4$? $4^1/_2$ is $2^1/_3$? $4^7/_8$ is $3^1/_4$? $5^2/_3$ is $4^1/_3$?

10. $6^1/_8$ is $5^3/_4$? $5^1/_8$ is $4^3/_7$? 11 is $5^1/_2$? $5^9/_{10}$ is $2^3/_5$? $7^1/_2$ is $6^1/_3$?

159. In problems such as the preceding, the results may be reached by taking the number representing the *part* for the *numerator* and the number representing the *whole* for the *denominator* of a fraction, and reducing as suggested in Art. 157, or, by dividing the former by the latter. Thus:

1. What part of 4 is $1^2/_3$? **Solution:** $\dfrac{1^2/_3 \times 3}{4 \times 3} = \dfrac{5}{12}$

2. What part is $5^3/_4$ of $7^5/_6$? **Solution:** $\dfrac{5^3/_4 \times 12}{7^5/_6 \times 12} = \dfrac{69}{94}$

SLATE EXERCISES.

What part of

3. 42 is $^3/_7$? 14 is $^4/_7$? 30 is $^1/_7$? 38 is $^1/_2$? 45 is $^5/_9$?

4. 48 is $^5/_6$? 33 is $^4/_{11}$? 52 is $^5/_{13}$? 48 is $^3/_4$? 55 is $^{11}/_{12}$?

5. $^4/_9$ is $^1/_3$? 5 is $2^1/_5$? $4^1/_{12}$ is $3^1/_6$? $1^5/_9$ is $1^1/_7$? $5^1/_2$ is $4^1/_3$?

6. $4^4/_5$ is $2^2/_{10}$? $7^1/_5$ is $4^1/_3$? $8^1/_9$ is $5^1/_7$? $8^2/_3$ is $7^1/_6$? $10^{10}/_{15}$ is $5^1/_3$?

7. 12 is $7^1/_4$? $12^1/_3$ is $6^1/_3$? $11^2/_5$ is $^1/_5$? $^1/_5$ is $^1/_{35}$? $^1/_{35}$ is $^1/_{70}$?

8. $^1/_3$ is $^1/_{12}$? $^1/_{24}$ is $^1/_{72}$? $^1/_{11}$ is $^1/_{121}$? $1^1/_{11}$ is $^3/_{77}$?

160. From a known fractional part of a number to find the number.

Example.—1. 6 is $^3/_4$ of what number?

Analysis.—If 6 is $^3/_4$ of a number, one fourth of the number is $^1/_3$ of $6 = 2$, and four fourths is 4 times $2 = 8$.

Example.—[illegible] $^{18}/_{19}$ is $^6/_7$ of what number [illegible]

Analysis.—If $^{18}/_{19}$ is $^6/_7$ of any number, $^1/_7$ of the number is $^1/_6$ of $^{18}/_{19}$ $= {^3/_{19}}$ of the number, and $^7/_7$ or the entire number is $7 \times {^3/_{19}} = {^{21}/_{19}} = 1^2/_{19}$.

3. $^5/_7$ is $^5/_8$ of what number?
4. 7 is $^3/_8$ of what number?
5. $4^1/_2$ is $^9/_{10}$ of what number?
6. $^2/_3$ is $^{11}/_{13}$ of what number?
7. $5^1/_2$ is $^1/_3$ of what number?
8. $6^2/_3$ is $^5/_6$ of what number?

Aliquot Parts.

161. An *aliquot part* of a number is any integer or mixed number that will exactly divide it without a remainder.

Aliquot Parts of a Dollar.

$50¢ = \$^1/_2$
$33^1/_3¢ = \$^1/_3$
$25¢ = \$^1/_4$
$20¢ = \$^1/_5$
$16^2/_3¢ = \$^1/_6$
$12^1/_2¢ = \$^1/_8$
$10¢ = \$^1/_{10}$
$8^1/_3¢ = \$^1/_{12}$
$6^1/_4¢ = \$^1/_{16}$
$5¢ = \$^1/_{20}$
$87^1/_2¢ = \$^7/_8$
$75¢ = \$^3/_4$
$62^1/_2¢ = \$^5/_8$
$60¢ = \$^3/_5$
$37^1/_2¢ = \$^3/_8$

162. To find the cost of a number of articles when the price is an aliquot part of a dollar.

Example.—1. What will 20 doz. eggs cost @ 25¢ a doz.?

Analysis.—At \$1 a doz., 20 doz. would cost \$20, but at 25¢ (= $^1/_4$ dol.) a doz., 20 doz. will cost $^1/_4$ of \$20, or \$5.

2. What is the cost of 28 readers @ 25¢? @ 50¢?
3. 144 lb. of beef @ $12^1/_2$¢? @ $16^2/_3$¢? @ $8^1/_3$¢?
4. 240 lb. of raisins @ 20¢? @ $16^2/_3$¢? @ $12^1/_2$¢?
5. 48 pr. of socks @ 50¢? @ $33^1/_3$¢? @ $37^1/_2$¢?
6. 160 lb. of sugar @ 5¢? @ $6^1/_4$¢? @ $8^1/_3$¢? @ $12^1/_2$¢?

7. 24 pr. of gloves @ $87\frac{1}{2}$¢ ? @ 75¢ ? @ $62\frac{1}{2}$¢ ?

8. 35 handkerchiefs @ 60¢ ? @ 40¢ ?

9. 16 baskets @ $37\frac{1}{2}$¢ ? @ $62\frac{1}{2}$¢ ? @ $87\frac{1}{2}$¢ ?

10. 12 pr. of slippers @ \$1.25 ? @ \$$1.33\frac{1}{3}$? @ \$$1.16\frac{2}{3}$?

11. 230 bu. of wheat @ $62\frac{1}{2}$¢ ? @ 75¢ ? @ $87\frac{1}{2}$¢ ?

12. 84 chairs @ [illegible].25 ? @ \$$1.33\frac{1}{3}$? @ \$$1.37\frac{1}{2}$?

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163. To find the number of articles that can be bought for a given sum, when the price of one is an aliquot part of a dollar.

13. At $33\frac{1}{3}$¢, how many lb. of butter can be bought for \$7 ?

Analysis.—If $33\frac{1}{3}$¢ will buy 1 lb., \$1 will buy 3 lb., and \$7 will buy 7 × 3 lb. = 21 lb.

Thus we have the convenient

***Rule.*—Multiply the number of articles that can be bought for \$1 by the number of dollars.**

14. At $16\frac{2}{3}$¢, how many books can be bought for \$9 ? At $33\frac{1}{3}$¢ ?

15. At 25¢, how many pr. of cuffs can be bought for \$7.50 ?

16. At $33\frac{1}{3}$¢, how many handkerchiefs can be bought for \$6 ? @ 50¢ ? @ 25¢ ?

17. For \$3.25, how many qt. of milk can be bought @ $6\frac{1}{4}$¢ ? @ 5¢ ? @ $8\frac{1}{3}$¢ ?

18. Find the cost of

7 thermometers	@	.50
9 thermometers	@	.75
5 pr. opera-glasses	@	\$$9.12\frac{1}{2}$
3 pr. roller-skates	@	\$1.75
72 scroll-saw blades	@	.20
13 pr. pincers	@	.35
3 magnets	@	.75
18 pocket compasses	@	\$$1.16\frac{2}{3}$
16 yards silk ribbon	@	$.87\frac{1}{2}$
$3\frac{1}{2}$ lb. copper-wire	@	$.62\frac{1}{2}$
$\frac{1}{2}$ lb. fine copper-wire	@	\$$1.87\frac{1}{2}$

19. Find the cost of

13 printing-presses	@	\$2.75
5 velocipedes	@	\$7.75
7 canoes	@	\$55.75
$3\frac{1}{4}$ bushels potatoes	@	.90
16 cwt. rice	@	\$$6.06\frac{1}{4}$
32 bushels wheat	@	\$$1.16\frac{2}{3}$
12 sacks salt	@	\$$1.33\frac{1}{3}$
9 lb. candy	@	$.37\frac{1}{2}$
3 qt. ice cream	@	.40
2 bicycles	@	\$97.50
64 lb. wool	@	$.37\frac{1}{2}$

Miscellaneous Examples.

ORAL EXERCISES.

1. Multiply the numerators of $^1/_2$, $^1/_3$, $^1/_6$, $^3/_8$, $^3/_4$, $^2/_3$, $^5/_6$, $^7/_8$, by 3; by 4; by 5; by 6. How do these multiplications affect the value of the fractions?

Note.—Frequent illustrations should be given with [illegible]rams and counters.

2. Divide the numerators of $^2/_4$, $^4/_8$, $^4/_{12}$, $^8/_{12}$, $^8/_{24}$, $^4/_{16}$, $^6/_{24}$, $^8/_{18}$, $^{12}/_{32}$, by 2. How are the values of the fractions affected by these divisions?

3. Multiply the denominators of $^1/_2$, $^2/_3$, $^5/_6$, $^3/_4$, $^5/_8$, $^7/_8$, by 4; by 5; by 10. Are the fractions increased or diminished by these multiplications?

4. Divide the denominators of $^4/_4$, $^4/_8$, $^4/_{12}$, $^8/_{12}$, $^8/_{24}$, $^6/_{24}$, $^8/_{32}$, by 2; by 4. How are the values of the fractions affected by dividing their denominators by integers?

5. State four ways in which the value of a fraction may be changed, and give examples to illustrate each.

6. What fraction is $^1/_4$ of $^{60}/_{97}$? $^1/_{10}$, $^1/_{15}$, $^1/_{12}$ of the same?

7. By how much is $^1/_3$ greater than $^1/_{12}$? Find the difference with the aid of slips of paper or parts of other objects. Tell what you do, and state the result.

8. Add together $^1/_3$, $^1/_4$, $^1/_5$, $^1/_6$, and $^1/_{15}$ of 60.

9. Which is the greater, $^5/_8$ of 40, or $^5/_6$ of 30? $^8/_9$ of 72 or $^9/_{11}$ of 77? (Illustrate with counters.)

10. How many times greater is $^1/_5$ than $^1/_{15}$? $^1/_6$ than $^1/_{24}$? $^2/_5$ than $^2/_{15}$? $^5/_6$ than $^5/_{24}$?

11. Add 3 to each of the terms of $^3/_7$, and tell how much the value expressed is increased or diminished.

12. Invert $^3/_7$ thus, $^7/_3$; add 3 to each term. By how much is the value of this fraction increased or diminished?

13. From 75 subtract $^7/_{15}$ of 75, and find $^2/_5$ of the remainder.

14. What is the difference between $^3/_5$ of $^5/_6$ and $^3/_4$ of $^2/_3$?

15. Sixteen is $^4/_5$ of what number? 14 is $^2/_3$ of what number?

16. A train starts from Indianapolis at $7^3/_4$ A. M.; it reaches Cincinnati $6^2/_3$ hours later. At what o'clock does it arrive?

17. Add together $^9/_2$ inches, $^3/_4$ foot, and $^1/_8$ yard.

18. I bought 16 oranges for 24¢. How much was that per dozen?

19. Man ordinarily spends $^1/_3$ of his time in sleep. How many hours at that rate does he sleep in a fortnight (14 days)?

20. If I read $^2/_5$ of a book in a day, how long at the same rate shall I be in reading the whole of it?

21. Eight times $^3/_4$ of $18 = ?$ 9 times $^2/_3$ of $16 = ?$ 7 times $^5/_9$ of $15 = ?$

22. I had $\$^3/_4$; spent $\$^1/_{10}$ for ink, $\$^1/_5$ for writing-paper, and $\$^1/_{20}$ for pens. How much money had I left?

23. Read the following fractions in the order of their value, beginning with the smallest: $^7/_4$, $^7/_6$, $^7/_{24}$, $^7/_8$, $^7/_{12}$.

24. Eighteen and two fifths yards are cut from a piece of cloth measuring $27^3/_5$ yd. What fraction of the piece remains?

25. Five ninths is $^2/_3$ of what number? $^7/_{12}$ is $^3/_7$ of what number?

26. How many copper wires $^1/_{30}$ of an inch in diameter must be laid side by side to cover $^1/_2$ inch? $^2/_3$, $^5/_6$, $^9/_{10}$ inch?

27. In an orchard $^1/_6$ of the trees are apple-trees, $^1/_{12}$ pear, $^1/_9$ plum, $^1/_3$ peach, and 22 are cherry-trees. How many in all?

28. Five little girls held a fair for the benefit of the Fresh-Air Fund. After paying out for expenses $^1/_{10}$ of the whole sum received, they contributed $36 to the Fund. How much did they receive?

29. Early June peas are 18¢ a can at retail. How much do I save per can by buying them at wholesale, $1.90 per doz.?

30. The sum of two numbers is $2^7/_8$. One of the numbers is $1^2/_3$. What is the other?

31. Johnny weeded $^1/_7$ of his garden on Monday; $^1/_6$ on Tuesday; $^1/_3$ on Wednesday, and in the remaining days of the week finished the task in equal portions. What part did he do each of those days?

32. Edith earns a cent for each extra $^1/_2$ hour she practices her music. Last week she earned \$$^1/_4$. How many extra half-hours did she practice?

33. A woman weaves 6 yards cloth in 2 days, of 12 hours each. What part is the work of 1 hour? If she is paid \$$^5/_8$ a yd., what are her day's wages?

34. A painter bought $18^3/_4$ quarts of turpentine at $^1/_6$ of a dollar per qt. He sold it at $^1/_4$ of a dollar per qt. What did it cost him, and what was his profit?

35. Divide the sum of $2^1/_3$ and $3^1/_2$ by their difference.

36. Twelve yards of goods $^3/_4$ yd. wide will make me a dress. How many yards will I need of silk that is $^1/_2$ yd. wide?

37. On the 4th of July Mr. Brown divided $^3/_5$ of \$4.00 among his children. To the eldest he gave $^1/_4$ of the $^3/_5$, and to each of the others 45¢. How many children had he?

38. John and Will together mow the lawn. John mows $^1/_4$ in 1 hour, and Will $^1/_6$. How long does it take both to mow it?

39. If a man can do $^5/_9$ of a piece of work in 4 days, in what time will he do the entire job?

Analysis.—If he does $^5/_9$ in 4 days, he will do $^1/_9$ in $^1/_5$ of $4 = ^4/_5$ day, and $^9/_9$, or the whole, in $9 \times ^4/_5 = ^{36}/_5 = 7\,^1/_5$ days.

40. If $^4/_{11}$ of an acre of ground yields 40 bushels of tomatoes, how many bushels per acre?

41. If $^3/_8$ of a bushel of Bermuda potatoes costs 90¢, what is the price per bushel?

42. If $^4/_7$ of a yard of velvet costs \$5, what does 1 yard cost?

43. Mr. Jackson sold $^7/_{13}$ interest in his shop for \$5,600. What was the whole business valued at?

44. If a horse can trot $^3/_8$ of a mile in 1 minute, in what time, at the same rate, can he trot 1 mile? $^5/_8$ of a mile?

45. If I had $^1/_3$ and $^1/_4$ more tulips in my garden I should have 57. How many have I now?

46. If a man earns \$$^1/_3$ per hour, a woman \$$^1/_5$, and a boy \$$^1/_{12}$, what do all receive for 1 hour's work?

47. How many hours must the boy work to receive an hour's wages of a man? How many the woman?

48. In what time, working together, can the woman and the boy earn an hour's wages of the man?

49. What number diminished by $^2/_7$ and $^5/_9$ of itself leaves a remainder of 30?

50. Seven tenths of a certain number less $^5/_8$ of it is 15. What is the number?

51. My age is $^3/_5$ of my brother's; his age is $^5/_{12}$ of father's, who is 72 years old. How old am I?

52. A plague carried off $^2/_9$ of a flock of sheep in one week, $^3/_7$ of the remainder the next, and 28 were left. What was the original number of sheep?

53. A contractor was to receive \$60,000 for a building, but forfeited $^1/_{40}$ that amount because it was not finished within the specified time. How much did he lose?

54. If 2 leaps of a dog are equal to 3 leaps of a hare, how many leaps of the dog are equal to 27 of the hare?

55. What number is reduced to 64, when $^8/_9$ of it are taken away?

SLATE EXERCISES.

1. My study measures $14^3/_4$ feet in length, and $12^2/_3$ feet in width. How many square feet in the floor?

2. A money-bag contains 37 half-dollars, 49 quarter-dollars, 37 twenty-cent pieces, 39 dimes, 63 nickels. How much money in all?

3. An employer pays $\$95\frac{1}{5}$ to his workmen, each one receiving $\$13\frac{3}{5}$. How many are there?

4. If 56 laborers earn each $\$\frac{1}{3}$ per hour, how much do they all earn in 6 days and 6 hours, reckoning 8 hours to the day?

5. Twenty-nine and four fifths yards were sold from a piece of cloth measuring $42\frac{3}{4}$ yd. What was the value of the remainder at $\$1\frac{17}{20}$ a yd.?

6. A merchant buys $52\frac{1}{2}$ bushels of beans at $\$1\frac{1}{2}$ a bu.; $35\frac{3}{5}$ bushels of peas at $\$1\frac{3}{5}$ a bu.; $28\frac{5}{8}$ bushels of cranberries at $\$2\frac{1}{3}$ a bu.—(1) Find the cost of each item. (2) Find the total cost, and the whole number of bushels.

7. He made a profit of $1\frac{1}{2}$¢ on every quart of beans, $2\frac{1}{3}$¢ on every qt. of peas, and $2\frac{1}{6}$¢ on every qt. of cranberries.—(1) Find the profit on each item. (2) Find the total profit.

8. A farm of $276\frac{11}{16}$ acres rents for $\$2013\frac{1}{2}$. What is the rent per acre?

9. I bought 45 government bonds at $\$105\frac{1}{2}$, and sold them at $\$106\frac{7}{8}$. Find the gain.

10. A house-painter earns $\$3\frac{3}{4}$ a day of 10 working hours. One week in which he worked extra time his pay amounted to \$25. How many extra hours did he work that week?

11. What will $7\frac{1}{4}$ yards of lace cost at $\$\frac{4}{5}$ per yd.? At $\$\frac{2}{3}$? At $\$\frac{5}{7}$? At $\$\frac{7}{12}$?

12. A person standing exactly under the equator is carried by the rotation of the earth 24,899 miles a day. How many miles is he carried in 1 hour, 2 h., 3 h., 5 h., 6 h., 8 h., 12 h.? (What part of a day is 1 hour? Do the work with as few figures as possible.)

13. If $7\frac{1}{2}$ lb. coffee cost $\$2\frac{1}{10}$, what will $11\frac{2}{7}$ lb. cost? $10\frac{5}{8}$ lb.? $4\frac{8}{9}$ lb.? $12\frac{1}{6}$ lb.?

14. Mr. A. left by will $\frac{3}{8}$ of his estate to his wife, $\frac{2}{5}$ of the remainder to his eldest son, $\frac{1}{3}$ of what was then left to his eldest daughter, and \$20,000 to each of his two other children. What was the value of the estate?

15. A grocer bought two tubs of butter, weighing together $70^{11}/_{12}$ lb. One tub when empty weighed $7^{1}/_{4}$ lb., and the other $8^{1}/_{3}$ lb. How much butter did he buy?

16. Find the sum and the difference of $(3^{3}/_{4} \div 5^{2}/_{5})$ and $(4^{2}/_{3} \div 5^{5}/_{8})$.

17. Find the sum and the difference of $(5^{4}/_{5} \times 3^{6}/_{7})$ and $(7^{3}/_{4} \times 3^{5}/_{6})$.

18. If it takes a workman $^{1}/_{6}$ of a day to do $^{2}/_{9}$ of a piece of work, how much of it can he do in $^{5}/_{8}$ of a day? How much in $^{3}/_{7}$ of a day?

19. If $^{3}/_{5}$ of an acre of land is sold for $\$45^{3}/_{20}$, what is the remainder worth at double the rate?

20. If $^{1}/_{15}$ of a box of merchandise is worth $\$7^{1}/_{8}$, what is $^{1}/_{5}$ worth?

21. A grocer mixes $57^{1}/_{2}$ lb. of tea, at $\$^{6}/_{10}$ a lb., with $42^{1}/_{2}$ lb. of tea at $\$^{7}/_{10}$ a lb. What is the value of a lb. of the mixture?

22. A farmer sold $^{5}/_{8}$ of his wheat at $\$1^{1}/_{10}$ a bushel, and received $\$796^{2}/_{5}$ for it. How many bushels did he sell, and how many did he have at first?

23. Mr. Hill, having $500 to pay expenses, made a journey that lasted 6 weeks. On reaching home he had $\$46^{4}/_{5}$ left.—(1) How much did he spend? (2) What was the average expenditure per week?

24. I bought a house and paid down $^{1}/_{3}$ of the price, and in one year thereafter I paid $^{2}/_{5}$ of the price. The two payments amounted to $43,780. What was the price of the house?

25. A clerk has a monthly income of $75, and spends $\$54^{2}/_{5}$ per month. How much does he save a year?

26. By how much would he have to diminish his expenses, per month, to save $\$20^{1}/_{2}$ per year more than he now does?

27. A laborer borrowed from his employer $\$66^{3}/_{20}$, agreeing to pay it by having $\$2^{45}/_{100}$ deducted from his wages every week. How many weeks at that rate did it take him to pay his debt?

28. If $^{2}/_{5}$ of 7 lb. of coffee costs $\$^{7}/_{8}$, how many lb. can be bought for $\$1^{23}/_{25}$? $\$2^{4}/_{5}$? $\$5^{3}/_{5}$?

29. What is the sum of the area of 5 fields, containing severally $93^{7}/_{8}$, $24^{3}/_{7}$, $86^{11}/_{56}$, $56^{17}/_{28}$, and $89^{9}/_{14}$ acres?

30. What is the cost of $23^{1}/_{2}$ lb. flour at $\$^{1}/_{20}$? $15^{3}/_{4}$ lb. oatmeal at $\$^{2}/_{25}$? $3^{2}/_{5}$ lb. raisins at $\$^{4}/_{25}$? $17^{1}/_{2}$ lb. nails at 4¢? 1 doz. fire-shovels at $12^{1}/_{2}$¢ apiece?

31. What number multiplied by $3^{7}/_{8}$ will give 2; what number divided by it will give $^{2}/_{3}$?

32. What number multiplied by $^{2}/_{3}$ of $11^{3}/_{4}$ will produce 1.

33. In a school of 100 pupils, of whom $^{3}/_{5}$ are boys, 7 boys and 4 girls are absent. What part of the boys are present? What part of the girls?

34. One third of the eighth part of what number is equal to $9^{1}/_{2}$?

35. How many cubic feet in a box $4^{1}/_{2}$ ft. long, $3^{1}/_{3}$ ft. wide, $7^{1}/_{2}$ ft. deep.? (See problems, page 103.)

36. If one faucet empties a cistern in 6 hours, and another in 9 h., in what time will both together empty it? What part of the contents will the two faucets discharge in 1 hour?

37. In what time will both empty it if the first begins to run after the second has run for 2 hours?

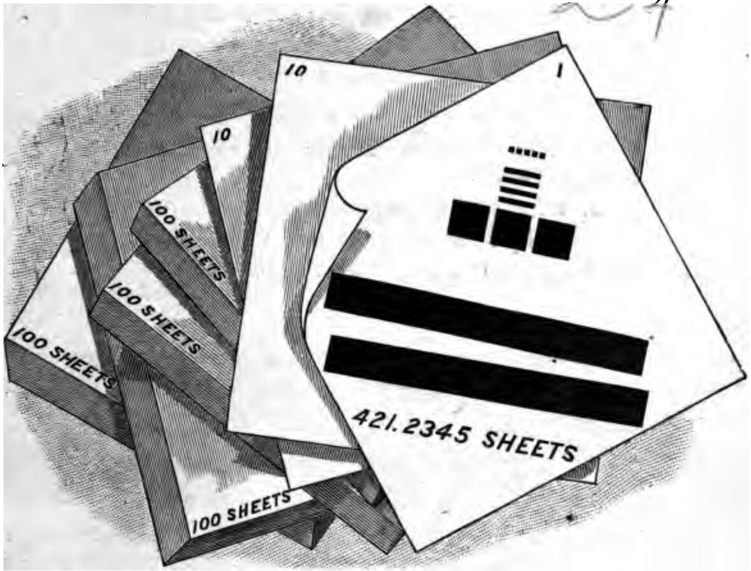
38. A can set the type for a certain book in 6 days, B in 8, C in 9, and D in 12 days. In what time can they do it working together? (What part will they all do in a day?)

39. How long must a room $4^{2}/_{5}$ yards wide be to contain as many square yards in the ceiling as a room $7^{1}/_{5}$ yards long and $5^{1}/_{2}$ yards wide?

40. I can walk 20 miles in 5 hours, and my friend can do it in 6 hours. Starting at the same time from points 20 miles apart and walking toward each other, how far are we apart in 1 hour, and in what time from starting would we meet?

CHAPTER X.

DECIMAL FRACTIONS.



164. The last chapter presented a mode of writing fractions in which the number of parts are indicated by one number and their names by another. This chapter shows how both the number and name of certain fractional parts may be represented by the decimal system.

Note.—Exercises on the following diagram are designed to familiarize the pupil with the relations of such parts. Bundles of jackstraws will also serve for illustration.

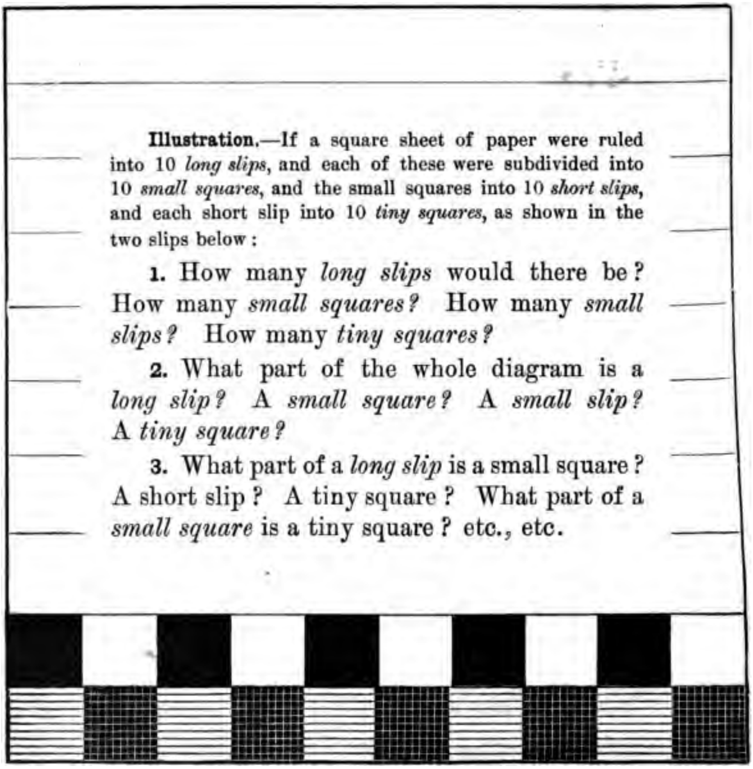


Illustration.—If a square sheet of paper were ruled into 10 *long slips*, and each of these were subdivided into 10 *small squares*, and the small squares into 10 *short slips*, and each short slip into 10 *tiny squares*, as shown in the two slips below:

1. How many *long slips* would there be? How many *small squares?* How many *small slips?* How many *tiny squares?*

2. What part of the whole diagram is a *long slip?* A *small square?* A *small slip?* A *tiny square?*

3. What part of a *long slip* is a small square? A short slip? A tiny square? What part of a *small square* is a tiny square? etc., etc.

Note.—The questions given above are only suggestive of exercises designed to make the pupil familiar with decimal parts and their relations.

165. The division of anything into ten equal parts, and the subdivision of these into ten smaller equal parts, and so on, are ***Decimal Divisions***, and the parts are ***Decimal Parts.***

Note.—The dime is a decimal part of a dollar, the cent a decimal part of a dime, the mill a decimal part of a cent.

166. A ***Decimal Fraction*** is one or more of the decimal parts of a unit.

Decimals expressed in Figures.

167. The first illustration (page 173) represents 421 sheets of paper, and 2 tenths, 3 hundredths, 4 thousandths, 5 ten thousandths of a sheet; and as each figure of 421 indicates by its place whether it represents units, tens, or hundreds, so the figures 2, 3, 4, and 5 may be made to indicate by their places whether they represent tenths, hundredths, thousandths, or ten-thousandths. But to show that they represent parts and not wholes, that they are *decimals* not *integers*, a point, called the ***decimal point*** (.), is placed before them, and the number is written thus: 421.2345.

168. The cipher is used in decimals, as in integers, to mark vacant places. Thus, if the two long slips were omitted in the illustration, the number represented would be expressed by 421.0345. If there were no long slips nor small squares it would be written 421.0045, etc., etc.

EXERCISES ON DIAGRAM.

1. How many sheets and how many and what parts of a sheet are represented by 4.2? 2.05? 3.82? .35? .23? 1.01? 2.71? .182? .19? 41.41? 3.00? .4321? 10.1? 7.15? 6.01? .101? 17.208? 15.001? 21.0021? .0053?

Give first the descriptive names of the parts, as long slips, small squares, etc., then use the proper arithmetical terms, tenths, hundredths, etc., thus: 4 sheets and 2 long slips, or, 4 sheets and 2 tenths of a sheet.

2. Illustrate by diagram, on slate or blackboard, what is meant by .01, by .25, by .35, by 3.7, by 1.3, by 2.004, etc.

3. Is there any difference in value between 6.7 and 6.70? Between 3.7 and 3.07? Between 5.16 and 50.16? Between .81 and .8100? (In stating the differences, tell what parts of the diagram are represented in each case.)

4. Tell how many long slips, small squares, etc., must be cut from a sheet of paper to have .357 of a sheet? To have .5642? To have .045? etc.

Without the aid of words, express in figures the number of sheets and parts of sheets described below, and read, using the proper decimal terms:

1. 207 sheets 7 long slips and 8 tiny squares; 3 small squares 5 short slips and 5 tiny squares.

2. 10 sheets and 1 tiny square; 75 sheets and 1 short slip; 6 sheets and 6 small squares.

3. 17 sheets 7 hundredths and 9 thousandths of a sheet; 13 sheets and 3 ten thousandths.

4. 24 sheets 3 tenths and 6 thousandths; 8 thousandths and 7 ten thousandths.

Note.—We can represent $^1/_3$, $^5/_6$, $^3/_7$, or other common fraction of a decimal part, by writing the common fraction after the decimal, thus: $.2\,^1/_3$ is read $2\,^1/_3$ tenths. $5\,^3/_7$ small squares would be expressed by $.05\,^3/_7$, which is read $5\,^3/_7$ hundredths. $.6\,^3/_7$ is 6 tenths and $^3/_7$ of a tenth.

Without the aid of words, express in figures:

1. 3 sheets $8\,^1/_2$ short slips; 7 sheets $6\,^1/_3$ small squares.

2. 13 sheets $8\,^2/_7$ slips; 23468 sheets $5\,^4/_9$ small squares.

3. 25 sheets $4\,^2/_3$ hundredths; 81 sheets $9\,^1/_9$ thousandths.

4. 86 sheets $5\,^5/_6$ tenths; 4000 and $7\,^3/_7$ ten thousandths.

5. Which has the greatest value, the $^1/_2$ in $2\,^1/_2$, $.2\,^1/_2$, or $.02\,^1/_2$?

Note.—We may represent entire decimal parts by fractions written in the common fractional form, thus: 3 small squares may be represented by $^3/_{100}$.

6. Tell how many whole sheets, long slips, etc., are represented by the following figures: $1\,^5/_{10}$, $2\,^7/_{100}$, $^8/_{10}$, $3\,^8/_{1000}$, $^3/_{100}$, $^7/_{1000}$, $20\,^9/_{10}$, $120\,^1/_{10}$, $99\,^9/_{100}$, $37\,^7/_{100}$, $4\,^3/_{10}$.

7. Write the following fractions in decimal form: $^{17}/_{10}$ (= 1.7), $^{131}/_{100}$, $^{3468}/_{100}$, $^{2426}/_{1000}$, $^{1769}/_{1000}$, $^{4432}/_{10}$, $^{1286}/_{1000}$, $^{316}/_{100}$, $5\,^7/_{10}$, $28\,^{29}/_{10000}$, $34\,^{36}/_{100}$, $5\,^{46}/_{1000}$, $^{71243}/_{10000}$, $104\,^5/_{10}$, $^{111}/_{1000}$, $^{19}/_{10}$.

Note.—Any fraction having for a denominator 10, 100, 1000, etc., is properly a decimal fraction, because it represents parts obtained by the division of the unit into tenths, tenths of tenths, etc., etc. But the term *decimal* is used alone only when there is no denominator expressed.

Definitions.

169. A ***Decimal point,*** or sign (.), is a period prefixed to a decimal to distinguish it from an integer.

170. A ***Pure Decimal*** consists of decimals only.

171. A ***Mixed Decimal*** is one that consists of an integer and a decimal.

172. A ***Complex Decimal*** is one consisting of a decimal with a common fraction annexed.

Decimal Table.

173. The following table will facilitate the learning of the several orders. The correspondence between the names of the places to the right and left of *units* should be noticed.

Table.

NAMES.	*Hundred-millions.*	*Ten-millions.*	**Millions.**	*Hundred-thousands.*	*Ten-thousands.*	**Thousands.**	*Hundreds.*	*Tens.*	**Units.**	*Tenths.*	*Hundredths.*	**Thousandths.**	*Ten-thousandths.*	*Hundred-thousandths.*	**Millionths.**	*Ten-millionths.*	*Hundred-millionths.*
ORDERS.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
	5	3	7	2	9	0	3	2	7.	0	3	2	1	4	5	1	6
	INTEGERS.									DECIMALS.							

Reading Decimals in Terms of the Lowest Order.

1. 2 long slips and 3 small squares, make how many small squares?

2. 2 tenths and 3 hundredths, make how many hundredths?

3. 5 long and 4 short slips, make how many short slips?

4. 2 tenths, 3 hundredths, and 4 thousandths, make how many thousandths?

5. 2 long slips, 3 small squares, 4 short slips, and 5 tiny squares = how many tiny squares?

6. 2 tenths, 3 hundredths, 4 thousandths, and 5 ten-thousandths = how many ten-thousandths?

Hence for reading decimals we have the

174. ***Rule.*—Read the decimal as if it were a whole number, and give it the name of the right hand order.**

Thus, .3567 is read 3567 ten-thousandths; .169 as 169 thousandths; .354789 as 354789 millionths.

ORAL EXERCISES.

1. Read, .1; .6; .9; .45; 11.4; 13.47; 51.67; 6.15; 8.24; 98.34; 100.1; 345693.71.

2. Read, 100.73; 27.02; 50.57; 6.67; 41.01; 120.03; 200.01.

3. Read, 1.111; .567; .004; 75.123; 3.004; 1.012; 6.953.

4. Read, 92.009; 9.00012; 13.8947; 57.625341; 1.06777893.

---

5. Read, pronouncing separately the order of each digit in the fractional parts: 61.43 **(61 and 4 tenths, 3 hundredths).**

10.9; 738.5423; 4.02; 5.063; 31.02803; 39.417356; 10.1324; 12.11; 26.103; 17.1101; 29.922; 30.87203456; 9.39485762.

---

6. Read the following as mixed decimals, that is, the units first then the decimal: **(Read 6.12 thus, six and 12 hundredths).**

14.013; 6.57; 3.0154; 46; 1044; 9.999; 20.02; 35.04; 46.34256; 50.148735; 83.4283; 87.87328; 7.5983.

7. Read the following as improper fractions, that is, read integer and fraction together as one number, giving to the whole the name of the lowest decimal order. **(Read 7.04 as 704 hundredths.)**

18.164; 516.2; 5.005; 29.092; 5.79; 13.579; 1357.9; 1.010; 263.4501; 63.4; 63.04; 63.004; 63.0004; 5.00013. **(The last is read, five hundred thousand thirteen hundred thousandths.)**

Suggestion.—Ask yourself whether it is true that 7.04 is equal to 704 hundredths. Turn to the illustration, on page 173, and study this out for yourself. How many small squares in 7 sheets of card-board? How many in 7 sheets and 4 small squares?

8. If .04 were written in the form of a common fraction, what would the numerator be? What the denominator? Answer like questions with regard to the decimals in exercises 1 and 2.

Note 1.—Observe that when the denominator is written, the decimal point and the ciphers preceding the first significant figure are omitted in the numerator: thus, $.03 = {}^{3}/_{100}$, not ${}^{.03}/_{100}$. (${}^{.03}/_{100}$ is equivalent to the complex fraction $\frac{{}^{3}/_{100}}{100}$.)

Note 2.—Observe, also, that the denominator of a decimal, when written, contains as many 0's as there are figures in the decimal.

Writing Decimals.

For the writing of decimals, the following rule will be found serviceable. Skill is to be obtained only by practice.

175. *Rule.*—Place the decimal point, then, after considering how many places are needed to give the last figure of the decimal its proper order; write each figure in the order to which it belongs.

Example.—Write 375 hundred thousandths.

Remembering that hundred thousandths is the fifth decimal order, and observing that 375 contains only three figures, we perceive that two orders must be filled with ciphers, thus: .00375.

SLATE EXERCISES.

Write in figures:

1. Three and fifteen hundredths; thirty-one thousandths.

2. One and one thousandth; twelve and fifteen hundredths.

3. One hundred twenty eight and seventeen thousandths.

4. Seventy-eight ten thousandths; seven hundredths.

5. Sixty-one hundred thousandths; ten and one ten thousandth.

6. Fifty-four thousand and fifty-four ten thousandths.

7. Five thousand seventy-five millionths.

Addition of Decimals.

176. ***Rule.*—Write the numbers to be added so that figures of the same order shall stand in the same column.**

Add as in integers, and place the decimal point in the sum directly under the decimal points in the numbers added.

Examples.—

1.	2.	3.	4.
3.523	.9374	.12	5.678934
23.42	13.21	5.2	2.16674
6.006	45.135	134.56	.00374
4.734	1.0006	42.03	17.00003

5. 6.6+77.77+888.888+26.742+1.2+5.401+.002=

6. 4.1535+.92+12.3472+.006+11.3+2.00046+9.07=

7. 100.2+59.012+8.+3.1205+69.+63.109+934563.4=

8. 604.1+.012+18.069+9.232+8.01+2.10004+3.05=

9. 10.901+12.+43.321986+.79342+4283.4132+6.7=

	10.	**11.**	**12.**	**13.**	**14.**	
15.	14.3	+2.348	+4.56	+17.01	+384.9000	=
16.	9.58	+8.71	+6.54	+ .004	+ 15.401	=
17.	73.374	+9.234	+3.042	+ 9.345	+ 3.789346	=
18.	1.583	+5.006	+7.1	+ 7.2003	+100.007384	=

Test the accuracy of your results. (See note, page 32.)

Applications.—**1.** Add the following sums of money: **$28.36, $108.09, $27.50, $1.30, $38.742, $387.655, $998.999, $3.27.**

2. Six marble blocks weigh respectively 5.73 cwt., 4.834 cwt., 7.938 cwt., 6.4 cwt., 15 cwt., and 387.1 cwt. Find the total weight.

3. A train on the Pennsylvania R. R. ran 56.3 miles in the first hour, 62.34 miles in the second, 59.247 in the third, 60.7304 in the fourth. How many miles altogether?

4. A draper bought 2 pieces of buckskin, each containing 56.34 yards; 2 pieces of rep, each containing 96.05 yards; and 1 piece of broadcloth, containing 27.2 yards. Find the number of yards in the 5 pieces.

Subtraction of Decimals.

177. ***Rule.*—Write the subtrahend under the minuend, so that figures of the same order shall stand in the same column.**

Subtract, as in integers, and place the decimal point in the remainder directly under the decimal points of the minuend and subtrahend.

Examples.—

1.	2.	3.	4.	5.
94.324	73.6	5.4	9.7	6.01
7.86	19.79	4.38	6.543	3.4

6.	7.	8.	9.
7384.02	9.004	3.28764	15.60003004
56.934	7.2043	1.00009	.794569376

10.	11.	12.	13.	14.
1.01	4.003	15.	70.	50009.
.09	2.006	6.3785	16.7345	5.0009

15. 8.452—3.1052=
16. 92.8245—9.86543=
17. .0052—.0041=
18. 3.004—.0097=
19. 121.12—8.943=
20. 423.4567382—413.05=
21. 73845.009—1.23456=
22. 9384.708—2.3457=
23. 342.5703—.1994=
24. 6534.70045—3.7634=
25. 897.309—3.1073=
26. 328.00019—6.0004=

Applications.—**1.** From a lot containing 10,000 □ yards, 437.296 □ yds. are sold. How large is the remaining part? (The sign □ is used for the word "square.")

2. From 17.256 tons of coal 5.625 tons were used. How much was left?

3. Mr. Smith's property amounted to $47,300.75 when he died. Accounts, to the amount of $340.95, were presented and paid. How much was left to the heirs?

4. The French meter is 39.37079 inches. How much longer than a yard is the meter?

5. Find the difference in height of two flag-staffs, the one measuring 38.75 ft., the other 53.9 ft.

6. Find the difference between .57 and .7; between eight hundred fifty-two ten-thousandths and 1.

Multiplication of Decimals.

Example.—1. Multiply .75 by 3. (Find 3 times .75.)

Process.

```
 .75
   3
----
2.25
```

Analysis.—3 times 5 hundredths = 15 hundredths = 1 tenth and 5 hundredths; 3 times 7 tenths = 21 tenths; 21 tenths + 1 tenth = 22 tenths = 2 units and 2 tenths.

Repeat the analysis, using the terms *small squares*, *long slips*, and *sheets*, respectively, for hundredths, tenths, and units.

The Multiplier a Decimal.

Example.—2. Multiply .75 by .3. (Find 3 tenths *of* 75 hundredths.)

Process.

```
 .75
  .3
----
.225
```

Analysis.—3 tenths of 5 hundredths = 15 thousandths = 1 hundredth and 5 thousandths; 3 tenths of 7 tenths = 21 hundredths; 21 hundredths + 1 hundredth = 22 hundredths = 2 tenths and 2 hundredths.

Repeat the analysis, using the descriptive terms *short slips*, etc. Thus, $^3/_{10}$ of 5 small squares = 15 short slips = 1 small square and 5 short slips, etc.

Example.—3. Multiply .75 by .03. (Find 3 hundredths of .75.)

Process.

```
  .75
  .03
-----
.0225
```

Analysis. — 3 hundredths of 5 hundredths = 15 ten thousandths (see diagram, page 174); 15 ten thousandths (tiny squares) = 1 thousandth and 5 ten thousandths; 3 hundredths of 7 tenths = 21 thousandths; 21 thousandths + 1 thousandth = 22 thousandths = 2 hundredths and 2 thousandths.

Repeat the analysis, using the descriptive terms *tiny squares*, etc.

178. Thus, we find that if the order of the multiplier is units, the order of the product is the same as that of the multiplicand. If the multiplier is tenths, the order of the product is *one degree* lower; if it is hundredths, the order of the product is *two degrees* lower, etc.

179. *Hence, in the product of two decimals there are as many decimal places as there are in the multiplicand, plus the number of decimal places in the multiplier.*

180. *Rule.*—**Multiply as in whole numbers, and from the right of the product point off as many figures for decimals as there are decimal figures in the multiplier and multiplicand together. If there be not so many figures in the product, supply the deficiency by prefixing ciphers.**

ORAL EXERCISES.

4.	5.	6.	7.	8.
.2×3=	14×.6=	.42×3=	.2×.3=	.2×.003=
.4×6=	13×.5=	.26×5=	.3×.6=	.9×.005=
.5×5=	15×.4=	.31×7=	.4×.5=	.5×.008=
.7×2=	17×.3=	.02×9=	.5×.2=	.7×.004=
.8×4=	16×.2=	.63×8=	.6×.4=	.3×.007=

SLATE EXERCISES.

Note.—It is well for pupils to accustom themselves to *estimate results;* for instance, if it is required to multiply 5.65 by 7.001, they should be able to say at a glance that the product will be *about* 39, that is, a little more than 5 $^{1}/_{2}$ times 7.

9.	10.	11.	12.
.3×.093=	.5×.934=	.52×.213=	1.5×.3=
.8×.075=	.7×.825=	.73×.332=	2.4×.5=
.5×.084=	.9×.738=	.84×.252=	1.5×.7=
.9×.063=	.6×.225=	.95×.163=	3.2×.6=
.7×.052=	.3×.367=	.62×.421=	1.6×.9=

13. 736.045×.843
14. 93×.0067
15. 4.709×.7635
16. 84.008×100.001
17. 17827.032×8.754
18. .0009×.0543
19. 9.00134×8.004
20. .195×.00027
21. .1825×18.24
22. .75×.30052
23. 84.008×1000.4
24. 258.01×3030.1
25. .98×36.0007
26. .357×88345.4
27. 28.601×3.425

28. Multiply .004; .71; .70014; 1.04 by .0091.
29. " .05; .17; .999; .7534 by .0008.
30. " 1000; 100; .001; .64; .01 by 2.847.

Applications.—**1.** My age is 1.075 times my brother's; if he is 30, how old am I? If he is 25, how old am I?

2. What is the area of a lot which is 9.34 yd. wide and 48.5 yd. deep? (How many square yards in it?)

3. Find the area of a field .876 miles by .0056 miles?

4. At $5.87 per acre, what is the rent of a farm of 47.9 acres?

5. If I buy 2 cwt. 66 lb. sugar at $13.09 per cwt., and sell it at $.12 per lb., what do I gain or lose on the whole?

Division of Decimals.

Example.—1. How many times .18 in 54?

Process.

```
.18)54.00
    -----
     300.
```

Analysis.—In 54 units there are 5400 hundredths, and 18 hundredths are contained 300 times in 5400 hundredths.

Illustration.—18 of the small paper squares represented on page 174 can be taken 300 times from 54 sheets.

2. How many times .18 in 5.4?

Process.

```
.18)5.40
    ----
     30.
```

Analysis.—In 5.4 there are 54 tenths = 540 hundredths. In 540 hundredths, 18 hundredths is contained 30 times.

Illustration.—Show that 18 small squares are contained 30 times in 5 sheets 4 long slips.

3. How many times .18 in .54? *Ans.*, 3.

4. How many times .018 in 54? In 5.4? In .54?

181. Let it be observed that in every case the dividend must be reduced to an order at least as low as that of the divisor. Evidently, if we are to ascertain how many times 18 short slips there are in any number of sheets, long slips, or small squares, we must first ascertain how many short slips there are. Hence, in division of decimals, there must always be as many decimal places in the dividend as in the divisor.

5. How many times 1.08 in .05778?

Process.

```
          .0535
1.08).05|778
      540-
       378
       324
        540
        540
```

Explanation.—Beginning with tenths, we count off as many decimal places in the dividend as there are in the divisor, and separate them from the places to the right by a short vertical line. This marks the point below which no integer can be obtained in the quotient (no quantity can be contained any whole number of times in a quantity less than itself). Here also the decimal places must begin, for, though one tenth of the divisor be not contained in the next partial dividend, the place must be marked by a cipher in order that figures of lower orders may have their proper places.

182. ***Rule*** **—1. Annex ciphers to the dividend, if necessary, till the right hand order is the same as that of the right hand figure of the divisor.**

2. Divide as in simple division. Place the decimal point immediately before the quotient figure that is obtained from the order of the dividend next lower than the lowest order of the divisor.

Note.—There must always be as many decimal places in the quotient as there are in the dividend more than in the divisor.

ORAL EXERCISES.

1.	2.	3.	4.
.4÷8=	2÷.5=	.64÷.08=	.16÷.8=
.6÷5=	5÷.3=	.49÷.07=	.14÷.7=
.8÷4=	4÷.6=	.36÷.03=	.08÷.4=
.2÷2=	5÷.7=	.84÷.04=	.09÷.3=

SLATE EXERCISES.

1. 7.32÷6
2. 123÷6
3. 127÷6
4. 4÷.008
5. 4.5÷67.8
6. .96÷32
7. .16÷.4
8. .58÷3.1
9. .6368÷8
10. 1÷.0025
11. 1÷.0037
12. 10÷.001
13. .5÷1000
14. 45.98÷10
15. 1000÷.5
16. 1000÷.09
17. .0045÷9
18. .03÷1.004
19. .0375÷.03
20. 7986÷3.75

21. 1.6875÷25
22. 134.25÷7.5
23. .0456÷.04
24. 733.26÷33
25. 1139÷9250
26. 789.7÷1000
27. 1.6875÷6.75
28. 2.0005÷7.24
29. 128.175÷7.5
30. 7.024÷2.0005
31. 604.56÷1000
32. 1220.674÷19
33. 144.6955÷8.5
34. 12.345÷.00015
35. 15.63386÷4.367

36. 549.9025÷2.345
37. 994.8015÷22.33
38. 600.2623÷66.77
39. 7.006652÷1.234
40. 1220.674÷64.246
41. .0013409÷.583
42. .0000026÷.004
43. 5941.8623÷66.77
44. 37.873565÷8.765
45. .0897688÷.0202
46. 245.8677645÷405
47. 20.34407403÷.21
48. 12345.4321÷111.11
49. 1.33709774÷.11111
50. 72.01440072÷8.0008

Applications.—1. The circumference of a circle is 3.14 times the length of the diameter. Find the diameter of a circle whose circumference is 51.339 yd.?

2. The area of a rectangle is 3414.012 □ yd., its width is 125.7 yd. What is its length?

3. 56.325 cwt. of certain goods cost $49.45335; what is the cost of 1 cwt.? Of 1 pound?

4. 36.35 yd. of cloth cost $117.95; what does 1 yd. cost at the same rate?

38 39 40

Reducing Common Fractions to Decimals and Decimals to Common Fractions.

Exercises on Diagram, page 174.

Express in decimals and also in lowest terms of common fractions the parts of the diagram

1. In 2, 3, 4, etc., long slips.
2. In 8, 25, 32, 20, 75 small squares.
3. In 2, 8, 14, 25, 125, 175 short slips.
4. In 8, 16, 32, 125, 1875, 625, 3125 tiny squares.

183. Changing Decimals to Common Fractions.—5. Express .6 in the lowest terms of a common fraction.

Process.—$.6 = {}^6/_{10} = {}^3/_5$.

6. Express .4, .8, .16, .72, .75, .375, .875, .4375, .04, .0016 in lowest terms of common fractions.

Note.—The learner will be able to write out his own rule for the foregoing process.

7. Express in integers and common fractions 1.2, 15.25, 8.6.

8. Express $.4^2/_3$ in the lowest terms of a common fraction.

Process.—$.4^2/_3 = \frac{4^2/_3}{10} = {}^{14}/_{30} = {}^7/_{15}$. (See Art. 156.)

9. In like manner find the equivalents of $.3^4/_5$, $.23^3/_8$, $.7^9/_{16}$, $.324^1/_7$ in common fractions.

7647
60.0
7
30
10
160
147
130
120
10

184. Changing Common Fractions to Decimals.—Any fractional part of an object must contain a like part of the decimal divisions of the object.

Thus, $^1/_2$ the diagram, page 174, contains $^1/_2$ of ten long slips = 5 long slips = .5; 50 small squares = .50 hundredths, etc., etc.

$^1/_4$ of the diagram contains $^1/_4$ of the decimal divisions, as: $2^1/_2$ long slips = $.2^1/_2$; or, 25 small squares = .25 of the diagram.

$^3/_8$ of the diagram contains $^3/_8$ of the long slips. $^3/_8$ of 10 long slips = $3^3/_4$ long slips = $.3^3/_4$. $^3/_8$ of 100 small squares = $37^1/_2$ small squares = $.37^1/_2$; and $^3/_8$ of 1000 short slips = 375 short slips = .375 of the diagram.

3|10\1000

185. Hence, to convert a common into a decimal fraction, we take such part of the decimal divisions of the unit as is indicated by the common fraction.

10. Find decimals equivalent to the common fractions, $^{2}/_{5}$, $^{3}/_{5}$, $^{3}/_{8}$, $^{9}/_{16}$, $^{11}/_{32}$, $^{12}/_{25}$, $^{21}/_{32}$, $^{8}/_{125}$, $^{27}/_{64}$, $^{101}/_{125}$, $^{333}/_{625}$.

11. Write in integers and decimals equivalents for $3\,^{1}/_{2}$, $563\,^{3}/_{40}$, $5\,^{1}/_{4}$, $7\,^{1}/_{8}$, $9\,^{17}/_{25}$, $16\,^{4}/_{5}$.

12. Find equivalents for $^{1}/_{3}$, $^{2}/_{3}$, $^{5}/_{6}$, $^{8}/_{9}$, $^{3}/_{7}$, $^{5}/_{12}$, $^{9}/_{14}$, $^{16}/_{21}$, $^{29}/_{30}$, $3\,^{2}/_{3}$, $7\,^{5}/_{6}$, $8\,^{5}/_{9}$, $4\,^{2}/_{7}$, $81\,^{3}/_{14}$, $9\,^{1}/_{9}$ in pure or mixed decimals.

Suggestion.—The question should be raised here, why it is that in Examples 10 and 11 all the common fractions are exactly reducible to decimals, while those in 12 are not. Thus the learner may discover for himself the condition under which exact decimal results are possible.

186. ***Rule.*—1. To reduce common fractions to decimals, annex ciphers to the numerator of the common fraction, divide by the denominator. Continue the process till the division is complete, or until the result is sufficiently exact.**

2. Point off as many decimal places in the quotient as there are decimal ciphers annexed to the numerator of the common fraction. If there be not so many places, ciphers must be prefixed to the significant figures to supply the deficiency.

Note.—The further the division is carried, the more exact is the result. In most cases sufficient accuracy is reached in the third or fourth place of decimals.

Repetends.—In the process of division, if a remainder is repeated, the figures of the quotient will be repeated in the same order as after its first occurrence.

187. A figure or set of figures thus repeated is called a ***Repeating*** or ***Circulating Decimal***, or simply a ***Repetend.***

188. The sign of a repetend is a dot (˙) written over the repeating figure, or a dot over the first and last figure, if it contains more than one.

Note.—At this point the pupil needs to learn no more of this subject than how to indicate a repetend when it occurs, and that he may discontinue the work of division on the first recurrence of any particular remainder. (See Appendix.)

Examples.—**1–7.** Reduce the following common fractions and indicate the repetends: $^{2}/_{7}$, $^{2}/_{9}$, $^{2}/_{11}$, $^{1}/_{3}$, $^{7}/_{13}$, $^{8}/_{15}$, $^{2}/_{13}$

SLATE EXERCISES.

Express equivalents in pure and mixed decimals:

1. $9.7\frac{1}{4}$, $7.7\frac{3}{5}$, $1.6\frac{1}{16}$.

2. $\$28\frac{13}{25}$, $\$17.07\frac{4}{5}$, $.053\frac{31}{32}$.

3. $15\frac{7}{20}$, $9.60\frac{7}{8}$, $105.00\frac{17}{25}$.

4. $\frac{33\frac{1}{3}}{4\frac{1}{5}}$, $\frac{17\frac{1}{8}}{7}$, $20.0\frac{3}{5}$.

5. $.0000\frac{2}{5} \times .9\frac{7}{8}$.

6. $10.111\frac{1}{3} \times .033$.

7. $5.009 \times .08\frac{1}{5}$.

8. $108\frac{1}{4} \times \frac{1}{3}$ of $\frac{9}{16}$.

9. Find the sum of $\frac{3}{4}$ and .54; the difference of $\frac{3}{4}$ and .54; the product of $\frac{3}{4}$ and .54; the quotient of $\frac{3}{4}$ divided by .54.

Find the sums of

10. $4\frac{3}{7}$, $524.2\frac{1}{7}$, $6.2\frac{3}{4}$, 7, and $.573\frac{3}{5}$.

11. $3\frac{4}{5}$ miles, $5\frac{5}{8}$ miles, 4.7 miles, 7.11 miles, and $99.9\frac{4}{5}$ miles.

12. 4.79 lb., $9\frac{4}{5}$ lb., $10\frac{9}{20}$ lb., $38.59\frac{1}{8}$ lb., 141.1 lb.

13. .125 rod, .1875 rod, $\frac{5}{8}$ rod, $.5\frac{5}{16}$ rod, $1.8\frac{3}{4}$ rod.

14. $1927.96\frac{13}{25}$ acres, $.00\frac{4}{25}$ a., 50.267 a., 1.709 a.

Find the differences between

15. $1.79\frac{1}{2}$ and $.777\frac{1}{8}$; $11.111\frac{1}{16}$ and $11.110\frac{1}{16}$.

16. $1.001\frac{1}{8}$ and $10.100\frac{1}{4}$; $7.9753\frac{2}{3}$ and $6.428104\frac{1}{5}$.

---

17. What number divided by 1.25 will give the product $11 \times 1.1 \times .001\frac{1}{10}$?

18. What was paid for 100 bbls. flour, each 196 lb., at $\$6.66\frac{2}{3}$ per 100 lb.? For 100 bbls. pork, each 200 lb., at $\$.08\frac{1}{4}$ a pound?

19. How many wagon loads in a freight car containing $2\frac{3}{16}$ tons sheet copper, 3.75 tons sheet lead, $5\frac{7}{8}$ tons sheet iron, 7.9375 tons tin plate, $1\frac{3}{8}$ tons being a wagon load?

20. From a sheet of lead weighing 1560.625 lb., circular discs were cut, weighing, respectively, $13\frac{1}{4}$ lb., $17\frac{1}{3}$ lb., 98.875 lb., 59.625 lb., $137\frac{1}{16}$ lb., $122\frac{5}{12}$ lb., $121\frac{1}{4}$ lb. What was the weight of the remnants (scraps)?

189. To find cost when number and price per hundred or thousand are given.

Per *C* is used for *per hundred* and per *M* for *per thousand.* (See page 18.)

Written Work.

```
 $3.60
  4.80
 28800
 1440
$17.2800
```

Example.—1. What is the cost of 480 lemons at $3.60 a hundred? 3 6 0

Explanation.—In 480 there are 4 hundred and 80 hundredths of a hundred; therefore we find 4 and 80 hundredths times the price of 1 hundred.

Note.—Ciphers at the right of a multiplicand or multiplier may be omitted in computation, inasmuch as they do not affect the value of the result. Hence the work may stand as at the right.

```
 $3.6
  4.8
  288
 144
$17.28
```

Written Work.

```
 $7.35
  17.3
  2205
 5145
 735
$127.155
```

2. What must be paid for 17300 bricks at $7.35 per M?

Explanation. — 17300 = 17.3 thousand; hence, to find the cost, at $7.35 per thousand, we multiply $7.35 by 17.3.

3. Find the cost of 7854 railroad ties at $95.50 a thousand.

4. Find the cost of 1478 feet of lumber at $45 per M.

5. Mr. Smith bought 50000 shingles at 70¢ a bundle of 250, and 38750 ft. of pine flooring at $18.75 a thousand. What did they cost?

6. We need 45350 bricks; the price being $6.90 a thousand, how much will they cost?

7. Mr. Wick bought 280 melons at $7.40 a hundred. What did they cost him?

8. Find the cost of 2750 laths at 45¢ per C; of 1950 pickets at $12 per M.

× 9. What is the cost of 1500 ft. of copper wire at $2.85 per hundred yards?

× 10. How much will the steel rails necessary to lay one mile of road cost at the rate of $49.30 for 100 ft. of rail? (5280 ft. = 1 mile.)

***Rule.*—Find the number of hundreds by pointing off two figures, and of thousands by pointing off three figures, on the right of the given number (representing the quantity), and by this multiply the price per hundred or thousand, as the case may be.**

---

190. To find the cost when the number of pounds and the price per ton (2000 lb.) are given.

Example.—1. What will a load of hay weighing 2386 pounds cost at $19.75 per ton?

Written Work.

2)2.386
1.193
19.75
5965
8351
10737
1193
23.56175

Explanation.—There are 2 thousand and 386 thousandths of a thousand pounds in the load, and one half as many, or 1.193, times 2000 pounds, or tons. Hence the value of the hay is 1.193 times $19.75, the price of 1 ton.

11–14. *Find the cost of*

3500 lb. of hay at $16 a ton.
4835 lb. of salt at $25 a ton.
9350 lb. of silver ore at $43.50 a ton.
380 lb. of straw at $9 a ton.

15. Find total freight charges on machinery shipped from New York to Buffalo in the following quantities, @ $^{7}/_{8}$¢ a ton per mile (see table, page 48):

15000 lb. locomotive castings.
31750 lb. flour-mill machinery.
17570 lb. pumping machinery.
49975 lb. saw- and planing-mill machinery.

16. What is the cost of 47.77 tons of iron rails @ 29^{1}/_{2}$ a ton. What would be the freight charges from Cleveland to Buffalo @ 1$^{1}/_{5}$¢ per ton for a mile?

17. Find cost of 978 tons Bessemer steel rails @ 40.33^{1}/_{3}$, freight being 1$^{5}/_{6}$¢ per ton for a mile, ordered in Cleveland and delivered in Jacksonville? (For distance, see page 48.)

18. What did I pay for 2975 pineapples at 11.87^{1}/_{2}$ per C?

19. What is the value of 9775 lb. ice at $6.75 a ton?

20. How many thousand cartridges can be bought for $855, there being 5000 in case, the cost of a case being $47.50?

***Rule.*—Multiply the price per ton by one half of the number of thousands of pounds** (number of tons).

Miscellaneous Examples.

1. A bricklayer earned $121.22 in 29 days; how much in 1 day?

2. 38 bales of cotton cost $3213.28; what is the cost of 1 bale?

3. The area of a garden in the form of a rectangle is 4133.64 sq. yd., its length is 76 yd. How wide is the garden? (To *find* the area we multiply the length by the width.)

4. 158 logs measure 3105.648 feet. What do they average?

5. What is the 87th part of 53.244 gallons? Of 53.244 qt.?

6. Divide $^4/_5$ of 8.236 by .138 of $^9/_{10}$.

7. What decimal of $2\,^1/_2$ yd. is $^1/_4$ yd.?

8. What part of 3 miles is $^1/_8$ of a mile? (Express the answer in decimals.)

9. Bought 3.75 cwt. beef at $.125 per lb.; find the total cost.

10. What number multiplied by 12 will produce .1728?

11. Divide the average of 3.079, 4.276, 5.60554 by .006.

12. If I walk 3.789 miles an hour, how far will my friend walk in 5 hours, if he walks only $^4/_5$ as fast as I do?

13. The French standard of measure, the meter, is 39.37 inches long; how many meters in 1132.134 yards?

14. How much carpet 1.5 yd. wide will cover a floor 22.5 by 19.5 ft.? (How many widths, the carpet being laid from end to end of the room?)

15. A regiment of 550 men has on its sick-list .02 of the number. How many men are fit for service?

16. What decimal fraction, multiplied by $^2/_5$ of $7\,^4/_5$ gives $^1/_4$ of $^2/_5$ of $^7/_8$?

17. The difference between two numbers is $17\,^{11}/_{24}$; the greater number is $25\,^1/_9$, what is the smaller number? (Answer in decimals.)

18. A bar of iron 8 inches square and 1 foot long weighs 216.336 lb., what is the total weight of 5 pieces respectively 3, 4, 5, 6, and 7 ft. long? (Only one multiplication necessary.)

19. What must the dividend be if the divisor is 38.125 and the quotient 5.25 ?

20. What number must be multiplied by 7 to make $^3/_4$? To make $^{11}/_{12}$? To make $2\,^1/_2$? (Solve each by decimals.)

21. Find the price of an ounce avoirdupois, if a lb. costs $.176 ; $.2475.

22. The water that will fill a can which is exactly one foot long, wide, and deep is 997.7 oz., or 62.356 pounds; hammered silver is 10.511 times heavier than an equal bulk of water. What is the weight of $2\,^1/_3$ cubic feet of the silver ?

23. After selling 78.38 acres of his land, a farmer had $198.6\,^3/_4$ acres remaining. How many acres did he have at first ?

24. Increase $^1/_4$ of 7.2 by $^3/_5$ of 6.5, and subtract from the sum the product of 5 × 1.14.

25. Mr. Smith has loaned $62848 to different parties for $\$4\,^1/_2$ per year for every hundred. What is his income per year from these loans ?

26. If one yd. of calico is sold for $.08, how many yd. can be had for $\$6\,^2/_5$, $\$4\,^2/_5$, $\$5\,^1/_5$, $\$7\,^3/_5$, $\$9\,^3/_5$?

27. An agent collected $347.35, and received for the service 5¢ on every dollar collected. How much did he get ?

28. In a city of 240768 inhabitants, it was found that .125 of the number could not read, and only .875 of those able to read could write. How many were there who could not read ? Who could not write ?

29. How much must be paid for the use of $750 per year at $\$5\,^1/_2$ a hundred ? For $^1/_2$, $^1/_3$, $^1/_4$, $^1/_5$, $^1/_6$, $^1/_{10}$, $^1/_{12}$ year ? (Express results in decimals.)

30. The use of $750 cost me $37.50 a year. What did I pay per hundred ?

31. The use of $1200 for 10 years cost Mr. Lund $630. What did the use of $100 cost him per year ?

32. Mr. Smith paid \$45 a year at \$4.50 per hundred for the use of a certain sum. What was that sum?

33. Mr. Cain borrowed a sum of money at \$3.25 a hundred per year, and in 5 years paid \$162.50 for the use of it. How great a sum was it?

34. Find the cost of 6 gal. 3 qt. vinegar, at \$.125 a gal. (3 qt. = what part of a gal.?)

35. Find the cost of 16 gross 6 doz. lead-pencils, at 55¢ a doz. (A gross is 12 dozen.)

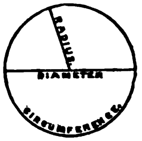
36. Find the cost of $13\frac{1}{4}$ yd. ribbon, at \$.2325 a yd.

37. Find the cost of 6.25 doz. cabbage-heads at 3¢ apiece.

38. Find the cost of 4 gross 10.5 doz. eggs, at \$$.22\frac{1}{2}$ a dozen.

39. If a railroad train runs 27.125 miles an hour, in what time will it run 303.6 miles?

Note.—The distance around a circle (the *circumference*) is very nearly 3.1416 times the distance across it through the center (the *diameter*). Let the pupil carefully measure the distance around and across a bushel or peck measure, or around and across a wagon wheel, or any other circle, and see if the distance around is not about $3\frac{1}{7}$ times the distance across it. ($\frac{1}{7}$ is very little greater than .1416.)



40. Find the circumference of a circle, the diameter of which is 4.2 yards.

41. The diameter of a wagon wheel is 40 in. How many yards will the wheel progress in turning 150 times?

42. Find the circumference of a circle, if the length of a radius is 3.75 in. If the diameter is 91.5 in. (Radius = $\frac{1}{2}$ Diameter.)

43. A block of gold measuring 1 in. long, wide, and thick (a cubic in.) weighs .7003 of a lb., how much does a cubic foot weigh? (See note, page 103.)

44. If I add the product of 11.111 by 22.02 to 33.033, and from this sum subtract 277.69721, what will the remainder be?

45. If you subtract the product of 5 by 31.565 from the sum of the two products 15 × .178 and 50.05 × 3.1, what will remain?

Bills and Accounts.

191. An ***Account*** is a record which may include services rendered, goods sold, or money paid by one person to another.

192. A ***Debtor*** is a person from whom a debt is due; a ***Creditor*** is a person to whom a debt is due.

193. A ***Bill*** is the creditor's written statement of the items in his account with the debtor.

194. Each item charged is called a ***debit;*** each item acknowledged as received is called a ***credit.***

195. The ***Balance*** of an account is the difference between the footing of the debits and the footing of the credits.

196. To ***receipt*** a bill is to write the creditor's name on the bill under the words ***"Received payment"*** or ***"Paid."***

A bill can be receipted only by the creditor or by a person authorized by him. In the latter case, the person receipting should write under the creditor's name "by" or "per," followed by his own name or initials. When a bill is paid by a promissory note or a due-bill, the fact may be stated after the words "Received payment."

197. To ***extend*** the items of an account is to write in the dollar and cent columns the cost of each article named at the price specified. To ***foot*** the several items is to write their sum at the bottom.

198. An ***Invoice*** is a detailed statement of the quantity, price, and description of goods sent to a purchaser or agent at one time. It includes also all charges, as for packing, cartage, insurance, etc.

The following signs and abbreviations are commonly used in business:

℀, account.
Acc't, account.
Am't, amount.
@, at.
Bal., balance.
Bo't, bought.
Co., company.
C. O. D., collect on delivery.
Cr., credit, creditor.
Do. or ", the same.
Dr., debit, debtor.
Fr't, freight.
Inst., this month.
Int., interest.
Mdse., merchandise.
Pay't, payment.
P'd, paid.
Rec'd, received.

Let the following bills be neatly and carefully copied, extended, and footed, with pen and ink. They may also be used as materials for dictation exercises.

1. *New York, March 31, 1885.*

Mr. George N. Bell,

Bought of Prince & Morton.

Feb.	*7*	*3 1/2 lb. Butter*	*@ $.32*		
"	"	*2 doz. Eggs*	" *.35*		
"	"	*1 gal. Molasses*			*76*
"	"	*2 lb. Mixed Coffee*	" *.28*		
"	*21*	*25 lb. Lump Sugar*	" *.07*		
"	"	*1/2 bu. Potatoes*	" *1.50*		
Mar.	*2*	*3 lb. Cheese*	" *.18*		
"	*3*	*2 lb. Raisins*	" *.15*		
				$	

Rec'd Pay't,

Prince & Morton.

2. *Chicago, Aug. 1, 1884.*

Mr. James S. Cooley,

To Edward Willis, *Dr.*

July	*7*	*To 1 Cheese-Dish*			*50*
"	"	" *1/2 doz. Butter-Plates*	*@ $1.50*		
"	"	" *1 1/2 doz. Dinner-Plates*	" *2.50*		
"	*9*	" *3 Candlesticks*	" *.25*		
"	*10*	" *5 Pitchers*	" *.75*		
"	"	" *1 1/2 doz. Cups and Saucers*	" *1.25*		
"	"	" *1 1/2 doz. Forks*	" *4.50*		
"	"	" *1 1/2 doz. Knives*	" *4.75*		
				$	

Rec'd Pay't,

Edward Willis.

3. *Cleveland, Jan. 11, 1885.*

MR. J. P. KINGSLEY,

1884. *Bo't of* ADAM JOHNSON.

Nov.	*3*	*9 yd. Cashmere*	*@ $.75*		
"	"	*1/4 yd. Velvet*	" *1.50*		
"	*5*	*6 yd. Lawn*	" *.12 1/2*		
"	*7*	*1 1/2 yd. Silesia*	" *.20*		
"	"	*3/4 yd. Cashmere*	" *.65*		
				$	

Rec'd Pay't,

ADAM JOHNSON,
By W. WRIGHT.

4. *Knoxville, Tenn., Aug. 15, 1885.*

MR. GEORGE CURTISS,

1885. *In acc't with* JAMES ARDEN AND COMPANY.

		DR.			
July	*3*	*To 1 Plow*		*$15*	*00*
"	"	" *2 Extra Plowshares*	*@ $1.50*		
"	"	" *1 1/2 lb. Powder*	" *.85*		
"	"	" *3 lb. Shot*	" *.11*		
"	*6*	" *3 Hoes*	" *.90*		
"	"	" *5 Rakes*	" *.95*		
				$	
		CR.			
July	*15*	*By 20 lb. Butter*	*@ $.13*		
"	"	" *3 doz. Eggs*	" *.18*		
"	"	" *10 bu. Potatoes*	" *.80*		
		Balance,		$	

Rec'd pay't,

JAMES ARDEN & CO.

5. *Atlanta, Ga., Apr. 14, 1882.*

MR. GEORGE READE,

To J. V. CAMP, *Dr.*

Apr.	6	*Repairing House as per contract*		$50	
"	7	*1 1/2 hours' labor*	@ $.32 1/2		
"	"	*12 ft. Clear Lumber*	" .04		
"	"	*2 lb. Nails*	" .05		
"	8	*10 hours' labor*	" .32 1/2		
"	"	*11 ft. Lumber*	" .03 1/2		
"	"	*Cartage*			25
"	"	*3 lb. Nails*	" .05		
				$	

Paid,

J. V. CAMP.

6. *Omaha, Neb., Sep. 3, 1882.*

MR. GEORGE HURLBURT,

1882. *In account with* WM. POWERS.

Aug.	17	*To 16 yd. Frieze with Inlay*	@ $.25		
"	"	" *Hanging 14 yd. Border*	" .02		
"	"	" *11 Rolls Paper*	" .37		
"	"	" *Hanging 11 Rolls Paper*	" .25		
"	19	" *Painting Kitchen*		$13	00
"	"	" *26 1/2 yd. Painting*	" .20		
"	21	" *27 yd. Painting in Hall*	" .25		
"	"	" *18 yd. Paper*	" .12		
"	"	" *6 hours Kalsomining*	" .30		
"	"	" *3 yd. Velvet Paper*	" .40		
				$	

Rec'd Pay't by note at 30 d.,

WM. POWERS.

Rule paper in proper form, and make out bills for the following transactions:

7. Mrs. Cole bought of E. P. Dale, of Boston, Feb. 5, 1884, 2 cans of String Beans, @ 10¢; $\frac{1}{2}$ bu. Potatoes, @ $1.00; 2 lb. Tea, @ 60¢; Feb. 9, 1 lb. Crackers, 20¢; 1 doz. Eggs, 32¢; 7 lb. Graham Flour, @ 4¢; Feb. 16, 3 cans Tomatoes, @ 12¢; 4 lb. Prunes, @ 16¢; 1 doz. Oranges, 50¢. Receipt the bill as clerk for Mr. Dale.

8. Charles Martin bought of Joseph A. Snow, of Pittsburg, Pa., Feb. 2, 1884, $2\frac{3}{4}$ lb. Mutton Chops, @ 22¢; $\frac{1}{2}$ pk. Apples, @ 40¢; 6 lb. Beef, @ 20¢; Feb. 6, $\frac{1}{2}$ pk. Sweet Potatoes, @ 30¢; 2 bunches Lettuce, @ 12¢; 2 qt. Turnips, @ 5¢; Chicken, $4\frac{1}{2}$ lb., @ 20¢; Feb. 9, 2 lb. Steak, @ 25¢; $\frac{1}{2}$ pk. Apples, @ 70¢; 1 qt. Onions, 10¢; Feb. 16, $7\frac{3}{4}$ lb. Beef, @ 20¢; 2 cans of Peas, @ 18¢; $\frac{1}{2}$ doz. Oranges, @ 50¢; Feb. 20, $9\frac{1}{2}$ lb. Ham, @ 18¢; Feb. 26, $2\frac{1}{2}$ lb. Lamb Chops, @ 22¢; 1 doz. Oranges, 50¢. Mr. Snow had bought of Mr. Martin 3 pt. of Cream, @ 12¢ a qt., daily through the month. Make out a receipted bill, using Bill 4 (page 196) as a model.

9. Alfred E. Robie bought of John Turner, of New Haven, Conn., April 2, 1885, $2\frac{1}{2}$ lb. Sausage, @ 14¢; $\frac{1}{2}$ doz. Lemons, @ 25¢; 2 lb. Dried Apples, @ 10¢; Apr. 4, $3\frac{1}{4}$ lb. Veal Chops, @ 20¢; Apr. 9, $\frac{1}{2}$ pk. Spinach, @ 70¢; 2 lb. Mutton, @ 14¢; Apr. 14, $\frac{1}{2}$ pk. Apples, @ 70¢; 2 qt. Sweet Potatoes, @ 10¢; Apr. 18, $6\frac{3}{4}$ lb. Beef, @ 20¢; $\frac{1}{2}$ doz. Bananas, @ 40¢; 2 doz. Pickles, @ 7¢; 2 qt. Bermuda Onions, @ 20¢; Apr. 23, $3\frac{1}{2}$ lb. Steak, @ 22¢; Apr. 28, 2 lb. Rhubarb, @ 10¢; 3 bunches Radishes, @ 7¢.

10. Mrs. James Bird bought of John Burns, of New Orleans, La., the following articles: Feb. 17, 1883, $\frac{3}{4}$ doz. Linen Napkins, @ $1.75; $2\frac{1}{4}$ doz. Damask Towels, @ $4.50; 3 Bath Towels, @ $2.40 a doz.; Feb. 21, 1883, 2 Table-cloths, @ $5.50; 1 Piano-cover, @ $5.00; 7 yd. Cambric, @ $.12$\frac{1}{2}$; 2 pr. Lace Curtains, @ $2.50 a pair.

11. Robert M. Miles bought of Lane & Bowers, of Philadelphia, Pa., Nov. 21, 1885, 1 Suit for \$28; 3 Shirts, @ \$1.25; 1 pr. Shoes, \$5.50; 6 pr. Socks, @ 35¢; 1 Umbrella, \$2.50; 2 pr. Gloves, @ \$1.75; and 4 pr. Cuffs, @ 35¢. Payment was made by note at 3 months.

12. Mr. George Ross bought of Robert James, of Albany, N. Y., on Mar. 13, 1881, 60 yd. Brussels Carpet, @ \$.85; 40 yd. Moquette Carpet, @ \$1.55; 35 yd. Canton Matting, @ \$.55; 3 Curtain-poles, @ \$4.50; 3 pr. Nottingham Lace Curtains, @ \$5.50.

13. Albert Halsted, in ℀ with George Reese: Aug. 7, 1881, $1\frac{1}{4}$ days' work, @ \$3.25; 44 ft. Pine Lumber, @ \$.06$\frac{1}{2}$; 1 lb. Nails, \$.07; work on Bookcases as per contract, \$13.00; 65 ft. Pine Lumber, @ \$.06$\frac{1}{2}$; $\frac{3}{4}$ lb. Nails, @ \$.07. Cr. by cash, \$5.00.

14. Mr. Robert Holden, of Brooklyn, New York, bo't of Stanley, White & Co., of New York city, Mar. 11, 1884, 3 doz. 8 in. Thermometers on polished walnut, @ \$10; $1\frac{1}{2}$ doz. 8 in. Parlor Thermometers, @ \$4 apiece; 5 doz. tin-case Thermometers, @ \$5; 9 Aneroid Barometers, @ \$5; 15 pr. Opera-glasses, @ \$4.25; 3 Microscopes, @ \$15; 1 large first-class Microscope, \$350; 2 Amateur Photographic Cameras, @ \$25. Paid by note at 3 mo.

15. Mrs. H. R. Otis bo't of Richard Hayes, June 11, 1880, 1 pr. Ladies' Kid Button Shoes, \$6; June 13, 2 pr. Ladies' Patent-Leather Oxford Ties, @ \$4.50; 1 pr. Misses' Kid Button Shoes, \$3.50; 2 pr. Infants' Black Kid Button Shoes, soft soles, @ \$.45; 1 pr. Child's Pebble Spring-heel Button Shoes, \$2.

16. James R. Baldwin bo't of Robert Price, Dec. 19, 1884, 1 copy "Little Men," \$1.35; 1 "Modern Explorers," \$10; 1 "Three Vassar Girls in South America," \$1.30; 1 "Rose in Bloom" and "Eight Cousins," \$2; 13 vol. Shakespeare, @ \$1; 3 vol. "Diamond Edition Poetry," @ \$.90; 5 vol. "Companion Edition Poetry," @ \$1.25; 6 vol. Hawthorne, @ \$1.35; 1 vol. "Sports and Pastimes for American Boys," \$1.25.

Suggestions for Original Problems.

1. Pupils will find suggestions for original problems in the Miscellaneous Exercises; or, it may be required that they construct problems of their own after models dictated by the teacher.

2. Having obtained reliable information from parents and others in regard to prices, trade customs, etc., they can make out bills, and furnish items for bills to be made by the class.

3. They may draw diagrams showing the forms and dimensions of lots to be fenced, dictate the kinds of fences to be built, prices of boards, posts, labor, nails, etc., and require the whole cost. They may give, in like manner, the information necessary to reckon the cost of digging cellars, building walls, laying board, stone, or brick walks, etc., etc. Pupils may often obtain from each other such information as may be needed.

4. Let illustrations, like the one on page 174, be required, showing .33, 1.27, etc., etc., of given squares.

5. Let pupils obtain where they can, the data necessary to enable them to calculate the cost of papering, carpeting, plastering, the schoolroom.

6. Pupils who have a little constructive skill may make paper boxes, and require their classmates to calculate their contents—how many quarts of blackberries or vinegar they will contain, etc.

7. Try the experiment of ascertaining the height of some tall tree or steeple, by measuring the length of its shadow, and the length of the shadow cast at the same moment by a stick or post, the length of which above ground can be easily measured.

8. Give the dimensions of a pile or load of wood, and ask, How many cords? or of a wood-shed, and ask, How many cords can be piled in it? or the length of a pile of wood, and ask how high it must be to contain some required number of cords.

9. Give the dimensions of a box containing a gross of such crayons as are used at the blackboard, and ask the length and width of a case which will exactly contain a gross of such boxes.

CHAPTER XI.

PERCENTAGE.*

If we were told that Mr. Jones had lost 6 out of every 40 peach-trees in his orchard, and that Mr. Smith had lost 3 out of every 25 in his, we should have to think awhile to tell whose *rate of loss* was the greater; but if we were told that Mr. Jones had lost 15 out of every hundred, and Mr. Smith 12 out of every hundred, we could compare the rate of their respective losses at once, because we would have a common standard of comparison.

199. ***Per hundred*** means by the hundred. ***Per cent.*** is commonly used for the phrase per hundred when the numbers compared represent things of the same kind.

Thus, we say that money at interest is worth *3 per cent.*—that is, 3 cents per hundred cents, or $3 per $100; that a pupil misspells six per cent. of the words given him, that is at the rate of 6 words in 100 words; but we would say that flour is worth $3 per hundred (weight), not 3 per cent.

200. A ***Rate per cent.*** is a rate per hundred.

201. The sign ***%***, annexed to the rate, stands for the phrase per cent.

Thus, 6% is read, six per cent.; $^1/_3$% is read, $^1/_3$ of one per cent.; .05% is read, five hundredths of one per cent.

Illustration.—Make 375 marks in three lines of 100 each, and one line of 75, or arrange 375 counters in like manner. If, now, 8 marks in each full line (of 100), and 6 in the line of 75, be crossed off, 8% of the whole number will be crossed off. (6 from 75 is at the same rate as 8 from a hundred). 8% may be added to the number of marks by adding 8 marks to each full line and 6 to the line of 75. Other per cents. may be illustrated in like manner.

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* The brief treatment of percentage and interest here presented is designed to meet the wants of pupils who will be withdrawn from school before completing the study of arithmetic as treated of in the Second Part. Those who take up that book may omit the last sixteen pages of this.

202. Any per cent. of a number is equivalent to so many hundredths of the number, or to the corresponding common fractional part of it.

25 % of any number is equivalent to .25, or $^1/_4$ of it. 25 counters from each full line (see illustration, bottom of preceding page), and $18\,^3/_4$ from the line of 75, or $93\,^3/_4$ in all = 25 %, or $^1/_4$ of 375.

ORAL EXERCISES.

Give the decimal and also the common fractional parts that are equivalent to:

1. 10 % 20 % 30 %
2. 25 % 50 % 75 %
3. 4 % 40 % 60 %
4. 80 % 8 % 16 %
5. 14 % 12 % 15 %
6. 15 % 24 % 60 %

7. What decimal and what common fractional part of a number is equivalent to $8^1/_3\,\%$ of it?

Solution.

$$8^1/_3\,\% = .08^1/_3 \qquad .08^1/_3 = \frac{8^1/_3 \div 8^1/_3}{100 \div 8^1/_3} = {}^1/_{12}$$

Note.—Since $6\,^1/_4$ is $^1/_{16}$ and $8\,^1/_3$ is $^1/_{12}$ of 100, any multiple of $6\,^1/_4\,\%$ or of $8\,^1/_3\,\%$ may be expressed in the form of a simple fraction.

What decimal and what common fractional parts of numbers are equivalent to:

8. $2^1/_2\,\%$ $18^3/_4\,\%$ $31^1/_4\,\%$
9. $37^1/_2\,\%$ $43^3/_4\,\%$ $56^1/_4\,\%$
10. $62^1/_2\,\%$ $68^3/_4\,\%$ $81^1/_4\,\%$
11. $8^1/_8\,\%$ $16^2/_3\,\%$ $33^1/_3\,\%$
12. $41^2/_3\,\%$ $58^1/_3\,\%$ $66^2/_3\,\%$
13. $83^1/_2\,\%$ $91^2/_3\,\%$ $6^1/_4\,\%$
14. $87^1/_2\,\%$ $93^3/_4\,\%$ $6^2/_3\,\%$
15. $3^1/_3\,\%$ $13^1/_3\,\%$ $23^1/_3\,\%$

What per cents. of a number are equivalent to the following fractions of the number:

16. $^1/_2$ $^1/_3$ $^2/_3$ $^1/_4$ $^3/_4$
17. $^1/_5$ $^2/_5$ $^3/_5$ $^4/_5$ $^1/_6$
18. $^5/_6$ $^1/_7$ $^1/_8$ $^3/_8$ $^5/_8$
19. $^7/_8$ $^1/_9$ $^1/_{10}$ $^7/_{10}$ $^1/_{11}$
20. .04 .40 $.41^2/_3$ .14
21. .75 .075 $.91^2/_3$ $.12^1/_2$
22. $.00^1/_3$ $.187^1/_2$ $.83^1/_3$ $.18^3/_4$
23. .0075 .005 $.58^1/_3$ $.37^1/_2$

Definitions.

203. The result of taking any per cent. of a number is called a ***Percentage.***

204. The number of which a percentage is given or on which it is to be computed, is called the ***Base.***

205. The base plus the percentage is called the ***Amount.*** The base minus the percentage is called the ***Difference.***

Cases in Percentage.

206. There are three principal cases in percentage.

CASE I.—To find a required per cent. of a number.

Example.—1. What is $37\frac{1}{2}\%$ of 168?

Solutions.

First Method.

One per cent. of 168 = .01 of 168 = 1.68; hence, $37\frac{1}{2}\% = 37\frac{1}{2} \times 1.68 = 63$ *Ans.*

Second Method.

Or, $37\frac{1}{2}\%$ of a number $= \frac{3}{8}$ of it. Hence, $37\frac{1}{2}\%$ of $168 = \frac{3}{8}$ of 168. $\frac{1}{8}$ of 168 = 21; hence, $\frac{3}{8} = 63$ *Ans.*

EXERCISES.

Find

1. 20% of 35; of 90; of 160; of 985; of 350; of 600.

2. $66\frac{2}{3}\%$ of 36; of 69; of 87; of 150; of 281; of 3900.

207. To find any required per cent. of a number.

***Rule.*—Take as many hundredths of the number or the corresponding common fractional part of it.**

Applications.—1. If I buy a horse for $280, and sell it so as to gain 25%, how much do I get for it? How much do I gain?

2. Of 9282 pupils in the public schools of a city, $14\frac{2}{7}\%$ are found to be near-sighted. How many near-sighted ones? What per cent. and how many not near-sighted?

3. A merchant buys goods at 6¢, 9¢, 12¢, 16¢, 18¢, 21¢, 26¢, 30¢, 81¢, $1.65, $2.25 per yard. What should he ask for them so as to gain $33\frac{1}{3}\%$ on each? (To avoid fractions of a cent, he would probably mark the fourth 21¢, and the seventh 35¢.)

CASE II.—To find what per cent. one number is of another.

Example.—1. What per cent. of 168 is 63 ?

Solutions.

One per cent. of 168 is 1.68, and 63 is as many times 1% as 63 is times 1.68. $63 \div 1.68 = 37\,^1/_2$.

$37\,^1/_2$ times $1\% = 37\,^1/_2\%$ *Ans.*

Or, 63 is $^{63}/_{168} = {}^3/_8$ of 168.

But, since $^3/_8$ of a number $= 37\,^1/_2$ per cent. of it, 63 is $37\,^1/_2\%$ of 168. *Ans.* $37\,^1/_2\%$.

EXERCISES.

What per cent.

Of 50 is 25 ? 75 ? 125 ? 250 ? Of 200 is 9 ? 12 ? 15 ?

Find what per cent.

1. 75 is of 600.
2. 87 “ 232.
3. 85 “ 272.
4. 81 is of 108.
5. 63 “ 72.
6. 60 “ 192.
7. $66\,^2/_3$ is of 200.
8. $27\,^1/_2$ “ 90.
9. 45 “ 54.

10. Divide 1000 into any number of parts, as 250, 300, 125, 325. Find what per cent. each part is of the whole. What must the sum of the per cents. be? Why ?

208. To find what per cent. one number is of another.

***Rule.*—Divide the percentage by 1% of the base, or find what part the percentage is of the base, and convert the fractions thus found into the corresponding per cent.**

Applications.—1. A lot was purchased for $3360, and sold at a loss of $560. What per cent. was lost ? What per cent. of the cost of the lot was received ? How much was received for it ?

2. A boy buys one sled for 80¢, and sells it for 40¢ ; at another time he buys one for 40¢, and sells it for 80¢. What per cent. is lost or gained in each case ? If he buys a top for 5¢, and sells it at 4¢, what per cent. is lost ? If he buys for 4¢, and sells for 5¢, what per cent. is gained ?

3. Mr. B. pays $8.60 for the use of $215 for 1 year. What per cent. does he pay ?

4. Mr. Long pays $43.50 insurance on $5800 worth of property. What per cent. does he pay ?

CASE III.—To find a number from a given per cent. of it.

Example.—1. 80 is 25% of what number?

Solutions.

If 80 is 25% of a number, 1% of the number must be $^{1}/_{25}$ of 80 = 3.2, and 100% or the number itself must be 100 × 3.2 = 320 *Ans.*

Or, 25% of any number is equivalent to $^{1}/_{4}$ of it; hence, if 80 is 25% of a number, it must be equivalent to $^{1}/_{4}$ of the number. 80 is $^{1}/_{4}$ of 4 × 80 = 320 *Ans.*

EXERCISES.

Find the number of which

1. 45 is 5%
2. 18 is $^{2}/_{3}$%
3. 40 is $62^{1}/_{2}$%
4. $11^{1}/_{9}$ is $^{5}/_{9}$%
5. 38 is $3^{4}/_{5}$%
6. 81 is 9%
7. 100 is $66^{2}/_{3}$%
8. 16 is $33^{1}/_{3}$%
9. 201 is $^{1}/_{3}$%
10. 7 is $2^{1}/_{3}$%
11. 91 is 13%
12. 195 is 200%
13. 45 is $41^{2}/_{3}$
14. 621 is $37^{1}/_{2}$%
15. 14 is $87^{1}/_{2}$%

209. To find a number from a given per cent. of it.

Rule.—**Divide the percentage by the rate** (to find 1%), **and multiply the quotient by 100** (to find 100% or the whole number).

Applications.—1. A tenant pays $850 rent, which is 10% on the valuation of the house. What is its valuation?

2. A clerk saves $147, which is $43^{3}/_{4}$% of his salary. What is his salary?

3. A farmer sold 585 bushels of corn in the fall, and 280 in the spring, the two sales together being $62^{1}/_{2}$% of the entire crop. How much corn did his farm produce that year?

4. For how much is my barn insured, if the insurance costs me $62.50 at $1^{1}/_{4}$%?

5. I sell a piano for $440, and thus lose 12% on what I paid for it. What did it cost me?

6. If $5000 be divided into any three unequal parts, what will be the sum of the remainders if 5% be taken from each part? If 9% be added to each part?

What must be the selling price if a dealer buys

1. Calico at 8¢ a yard, and sells so as to gain 25 %? $12\frac{1}{2}$ %? $37\frac{1}{2}$ %?

2. A lion for $750, and sells it so as to lose 27 %? $33\frac{1}{3}$ %?

3. An iron furnace for $65,000, and sells so as to gain $16\frac{2}{3}$ %?

4. 25 doz. pocket-knives for $212.50, and sells at a loss of 20 %?

What per cent., and how much, does a dealer gain or lose if he buys

5. 1000 tons coal at $4.20, and sells it at $5 per ton?

6. 2500 yards sheeting at 6¢, and sells it at 8¢ per yard?

7. A house for $5200, and sells it for $4800?

8. A gold-mine for $750,000, and sells it for $1,000,000?

What per cent. of the cost, and how much, does a dealer receive if he buys

9. 350 acres of land at $12.75, and sells for $17 per acre?

10. 112 books for $56, and sells them at $62\frac{1}{2}$¢ apiece?

11. 3 horses for $375, and sells them at $$112\frac{1}{2}$ apiece?

12. 10,000 yd. of cloth at $2.75, and sells for $3.50 per yd.?

What must a dealer have paid for

13. 200 yd. silk, if he gained 25 % by selling at $1.25 per yd.?

14. A picture, if he loses $33\frac{1}{3}$ % by selling it for $1540?

15. A carriage, if he loses $43\frac{3}{4}$ % by selling it for $180?

16. 160 bbl. flour, if he gains 10 % by selling at $3.30 a bbl.?

17. A dealer, in selling a shawl for $24, gains 20 %. What % would he gain if he sold at $30?

18. A bankrupt's debts amount to $385,580. The total value of his property is $72,296.25. What % can he pay on his debts?

19. How much will the above mentioned bankrupt pay on a debt of $578.60? of $51,240?

20. If $5000 be divided into any number of parts, what will be the sum of the percentages if $5\frac{3}{4}$ % be taken of each part? What per cent. of $5000 will the sum be?

Trade Discount.

210. A ***Trade Discount*** is a deduction from the price of an article.

A ***Net Price*** is the list price minus the trade discount.

Note.—In some branches of business it is customary to have fixed price-lists or certain kinds of goods, and when a rise or fall of prices occurs, instead of hanging the prices directly, the discounts are changed.

Thus, one dollar may be given as the list price of an article, with discounts of 0 % and 5 %, which means that the article is sold for 38¢. (A discount of 60 % rom $1 leaves 40¢, and a discount of 5 % from 40¢ leaves 38¢.)

211. Each discount is made from the result of preceding discounts.

Find the net prices.:

	List prices.	Discounts.		List prices.	Discounts.
1.	$36	15 % and 10 %.	3.	90¢	40 % and 10 %.
2.	$28	25 % " 5 %.	4.	$4.80	60 % " 20 %.

5. Mr. Smith offers me a discount of 25 % and 10 %, Mr. Jones discount of 20 %, 10 %, and 5 %, on a bill of $500. How much s one offer better than the other?

Insurance.

212. ***Insurance*** is security against loss by fire, water, accident, etc.

213. ***Life Insurance*** is a contract for the payment of a specified sum at the eath of the insured or at the end of a specified time.

214. The ***Premium*** is the sum paid for insurance. It is usually computed t a given rate per cent. on the sum insured.

215. The ***Policy*** is the written contract between the insurer and the insured.

216. The ***Insurer*** is also called an ***Underwriter***, because his name is ritten under the policy (i. e., *signs* the contract).

Example.—1. A farmer insures his dwelling for $3500, at 1 %; nd his barn, with contents, for $2500, at 1½ %? Find the premium.

2. Find the sum insured if the premium is $165 at ⅞ %.

3. Find the rate, the premium being $10.50 on $1750 insured.

4. How much is paid annually on $3500 life insurance at 17.50 per $1000?

Commission, Brokerage, etc.

217. A person who transacts business for another is an ***Agent*** for that person. The person for whom the agent acts is the ***Principal***.

218. ***Agents*** are known by various names, as, ***Commission Merchants, Brokers, Collectors, Correspondents,*** etc., according to the nature of their business.

219. ***Commission*** is the allowance to an agent for transacting business for another. It is usually reckoned at a given rate per cent. on the sum paid for what is bought or received for what is sold.

The reckoning of commissions requires only a simple application of the processes learned in percentage.

Examples.—1. A commission merchant (one who receives and sells goods for another) sells 580 barrels of flour at $7 per barrel, and charges 5 % commission. What is his commission? What is his commission on 268 barrels of apples, sold at $1.37 $^1/_2$ per barrel?

2. A broker (one who merely makes bargains for another) sells 5582 lb. of sole-leather at 15¢ a pound, and charges 5 % commission. What is his commission?

3. I send $525 to an agent, requesting him to buy some books, and take his commission out of the money sent him. If he charges 5 % commission, how much can he expend for books?

Solution.—If allowed 5 % commission, he puts 5¢ in his pocket for every 100¢ he pays for books; hence, for every $1.05 sent him he buys one dollar's worth of books. $1.05 is contained 500 times in $525; hence, with $525 he can purchase $500 worth of books and take 5 % of $500 as his commission.

4. If a broker charges $71 for effecting a loan of $5680, what per cent. does he receive?

5. A commission merchant sells goods for $8795, and retains $368.30 to pay charges amounting to $86.50 and his own commission. What per cent. commission does he take?

6. An auctioneer sells a bureau for $6.50, a set of chairs (6) for 88¢ apiece, and a carpet containing 35 yards at 35¢ per yard. He charges 5% commission on the sales. What is the amount of his commission?

CHAPTER XII.

INTEREST.

220. ***Interest*** is compensation for the use of money.

We pay *rent* for the use of a house, *hire* for the use of a horse, *interest* for the use of money.

221. The ***Principal*** is the sum of money, for the use of which interest is paid.

222. The ***Rate of Interest*** is the rate per cent. allowed for the use of money for one year or other specified time.

Note.—Any given rate is understood to be the rate for one year, unless the time be specified; as, per month, per day, etc.

In this book, when no rate is mentioned, 6 % is understood. Thus, in the question, "What is the interest of $50 for 6 months?" 6 % *per annum* is understood.

223. ***Legal Interest*** is a rate fixed by law for cases in which no rate is specified in the agreement between the parties interested.

In most of the States a limit is fixed to the rate of interest which may be charged. Interest at a rate exceeding the limit allowed by law is termed ***usury,*** to which some penalty is usually attached.

224. The ***Amount*** is the sum of the principal and interest.

If we hire money we return it and pay interest for its use, as we return a hired horse and pay for its use.

225. ***Simple Interest*** is interest on the principal alone, and is payable with the principal.

226. In the common method of computing interest, 12 months of 30 days each, or 360 days, are reckoned as 1 year.

Note.—Though this method of reckoning time is not exact, it is the most common because it is the most convenient. In most of the States it is legalized by law.

ORAL EXERCISES.

1. What must be paid for the use of $800 for $1\frac{1}{2}$ years at 6%?

Analysis.—The interest of $800 for 1 year at 1% is $8, and for $1\frac{1}{2}$ years it is $1\frac{1}{2}$ times $8 = $12. At 6% it is 6 times $12 = $72 *Ans.*

2. Find the interest of $200 for 1 year at 4%; of $500 at $3\frac{1}{3}$%; of $800 at 6%; of $700 at 8%; of $1000 at 10%; of $400 at 5%. Find the interest of each for 6 months ($\frac{1}{2}$ year).

SLATE EXERCISES.

1. What is the interest of $160 at $6\frac{1}{4}$% for 3 years?

Analysis.—The interest of $160 at 1% for 1 year is $1.60; at $6\frac{1}{4}$% it is $6\frac{1}{4}$ times $1.60, or $10, and for 3 years it is 3 times $10, or $30 *Ans.*

Find the interest for one year of

2. $450 at $4\frac{1}{2}$%

3. $680 at $3\frac{1}{2}$%

4. $2630 at $4\frac{1}{2}$%

5. $4920 at 5%

6. $7428 at $5\frac{1}{2}$%

7. $9654 at 6%

8. What is the interest of $80 for 9 mo. at 5%? (9 mo. = $\frac{3}{4}$ yr.)

Find the interest of

9. $70 at $4\frac{1}{2}$% for 5 yr.

10. $84 at $4\frac{1}{6}$% for $2\frac{1}{2}$ yr.

11. $97 at $6\frac{1}{4}$% for $3\frac{1}{3}$ yr.

12. $228.50 for 5 mo. 18 d.

Solution.

$$\begin{array}{rl} \$2.285 & \text{Int. for 1 yr. at 1\%} \\ 6 & \\ \hline \$13.710 & \text{Int. for 1 yr. at 6\%} \\ 168 & \text{Days in 5 mo. 18 d.} \\ \hline 360)2303.280 & \\ \hline \$6.398 & \text{Int. for 5 mo. 18 d.} \end{array}$$

Analysis.—Since 360 d. are reckoned as 1 year, the interest for 168 d. is $\frac{168}{360}$ of the interest for 1 year. Hence, we multiply $13.71, the interest for 1 year, by 168, and divide the product by 360.

Cancellation.—To shorten the work, we may arrange all the factors of the dividend on the right side of a vertical line, and place the divisor on the left, and cancel all common factors.

$$\begin{array}{ccc|cc} & & & \$\cancel{2}.\cancel{2}\cancel{8}\cancel{5} & \$.457 \\ \cancel{5} & \cancel{6}\cancel{0} & \cancel{3}\cancel{6}\cancel{0} & \cancel{6} & \\ & & & \cancel{1}\cancel{6}\cancel{8} & 14 \end{array}$$

$$14 \times \$.457 = \$6.398 \textit{ Ans.}$$

227. The foregoing method of computing interest is such as would be adopted by any one acquainted with the elements of arithmetic, though he might not have had any instruction in "Interest" as taught in the books. It is therefore sometimes called the ***general method.*** But any one who wishes to become expert in computing interest without the aid of Tables should be able to use shorter methods.

The 60 Day Method.

228. At the rate of 6% per year, the rate for 2 months or 60 days is 1%, which is equal to .01 of the principal; and for 6 days it is $^1/_{10}$ of 1%, which is equal to .001 of the principal.

Example.—1. Find the interest of $562 for 2 mo. 6 d. Also of $675; $1085; $98.50; $788.25.

Solution of the first.

Int. of $562 for 2 mo. =	$5.62
" " " 6 d. =	.562
2 mo. 6 d..............	$6.182

229. Thus we see that when the rate is 6% we may find the interest of any principal for two months by removing the decimal point *two* places to the left, and for six days by removing it *three* places to the left, prefixing ciphers when needed. By taking such multiples and parts of these results as the given time requires, and adding them together, the interest may be found for any given time.

2. How much must be paid for the use of $125.25 for 117 days at 6% per annum? What will be the amount?

Complete Solution.

Principal, $125.25; Int. 60 d., 1.2525; Int. 6 d., .12525.

			Shorter Process.
	1 . 2525	Int. 60 d..................	1 . 253
	62625	" 30 d..................	627
	4175	" 20 d..................	418
	12525	" 6 d..................	125
	020875	" 1 d..................	021
	$2 . 442375	Int. 117 d.	2 . 444
Principal,	$125 . 25		$125 . 25
Amount,	$127 . 692375		$127 . 694

Notes.—1. The interest for 60 days and for 6 days written out as in the first line above will be found convenient for reference. A perpendicular line under the first decimal point will serve for other points, and help to avoid confusion.

2. Results will be exact enough if the work is carried to mills, as at the right.

3. What is the interest of $18.56 for 1 year 8 mo. 16 days?

4. What is the interest of $198 for 2 years 11 mo.?

5. What is the interest of $89.90 for 9 mo. 9 d.?

6. What is the interest of $385.50 for 3 yr. 8 mo. 28 d.?

Interest at other Rates than 6%.

From the interest of any sum at 6% the interest at other rates is readily found.

Example.—What is the interest of $339.75 for 1 yr. 5 mo. 23 d. at 6%? At 3%? 4%? 5%? 7%? 8%? 9%?

Interest at 6%.

Solution.

Prin., $329.75; Int. 2 mo. = $3.2975; Int. 6 d. = .32975.

			Shorter Process.
	$26.3800	Int. 1 yr. 4 mo. (8 × 2 mo.)............	$26.38
	1.64875	" 1 mo. ($^1/_2$ of 2 mo.)...............	1.649
	1.09917	" 20 d. ($^1/_3$ of 60 d.)...............	1.099
	.16488	" 3 d. ($^1/_2$ of 6 d.)................	.165
Ans.	$29.29280	Int. for 1 yr. 5 mo. 23 d. at 6%.	*Ans.* $29.293

Interest at other per cents.

From the interest at 6%, to obtain the interest of the same sum for the same time—

At 3% divide by 2	**At 7% add $^1/_6$**
" 4% subtract $^1/_3$	**" 8% " $^1/_3$**
" 5% " $^1/_6$	**" 9% " $^1/_2$**

SLATE EXERCISES.

Find the interest and amount:

	Principal.	Rate.	Time.		Principal.	Rate.	Time.
1.	$600,	3%,	4 yr. 7 mo. 18 d.	**4.**	$869.44,	6%,	8 yr. 4 mo. 17 d.
2.	$796,	4%,	5 yr. 10 mo. 6 d.	**5.**	$1126.56,	7%,	10 yr. 5 mo. 1 d.
3.	$976.28,	5%,	7 yr. 9 mo. 27 d.	**6.**	$1295.28,	8%,	13 yr. 4 mo. 29 d.

Here the pupil should be taught how to find the difference of time between two dates according to the first method, page 231, Part II.

Find the amount:

	Principal.	Rate.	Time.
7.	$542,	7%,	From 1872, Oct. 27, to 1880, May 12.
8.	$684,	8%,	" 1868, Sept. 19, to 1870, June 1.
9.	$960,	9%,	" 1872, Dec. 31, to 1879, Oct. 1.
10.	$1100,	4%,	" 1839, Jan. 1, to 1850, Dec. 20.
11.	$86.75,	7 $^1/_2$%	" April 1, 1883, to Jan. 12, 1884.
12.	$98.87,	5%	" Mar. 15, 1885, to May 15, 1887.
13.	$54.45,	3 $^1/_2$%	" Nov. 17, 1881, to Feb. 21, 1882.

To find Principal, Per Cent., or Time.

It is sometimes required to find the principal, or the rate %, or the time, two of these elements being given with the interest or amount, thus,

I. *What principal* will produce $7.20 interest in 2 years at 6%?

Solution.—***$1*** **at 6% will in 2 years produce 12¢ interest. To produce $7.20 interest will require as many times $1 as $7.20 is times 12¢. $7.20 is 60 times 12¢, and hence it will require 60 times $1 = $60 to produce $7.20 interest in 2 year at 6% per annum.**

What principal will amount to $67.20 in 2 years at 6%?

Solution.—***The amount of $1*** **at interest for 2 years at 6% will be $1.12, and it will require as many times $1, to be invested for the same time and at the same rate, to amount to $67.20, as $67.20 is times $1.12, which is 60. Hence it will require 60 times $1 to amount to $67.20 in 2 years at 6%. 60 × $1 = $60 *Ans.***

II. *At what rate %* will $60 produce $7.20 interest in 2 years?

Solution.—***At 1%*** **per annum $60 will produce $1.20 interest in 2 years, and it will require as many times 1% to produce $7.20 interest as $7.20 is times $1.20, which is 6. Hence it will require a rate of 6% for $60 to produce an interest of $7.20 in 2 years.**

III. *In what time* will $60 produce $7.20 interest at 6%?

Solution.—***In 1 year*** **$60 at 6% will produce $3.60 interest, and to produce $7.20 will require as many times 1 year as $7.20 is times $3.60, which is 2. Hence it will require 2 years for $60 to produce $7.20 interest at 6% per annum.**

Hence,

230. I. To find the Principal (interest, time, and rate % being given).

***Rule.*—Divide the given interest by the interest of $1 for the given time at the given rate %.**

231. II. To find the Principal (the amount, time, and rate being given).

***Rule.*—Divide the given amount by the amount of $1 for the given time at the given rate %.**

Note.—Observe the difference between Rules I and II.

232. III. To find the Rate % (principal, interest, and time being given).

***Rule.*—Divide the given interest by the interest of the principal at 1% for the given time.**

233. IV. To find the Time (principal, interest, and rate % being given).

***Rule.*—Divide the given interest by the interest of the principal for 1 year at the given rate %.**

ORAL EXERCISES.

1. What sum must I invest to yield $54 interest per year at 6%? To yield $90 in two years at 5%?

2. At what per cent. must I invest $600 to produce $72 interest in three years? To obtain $42 on $420 in two years?

3. For how long a time must I put $350 ~~at 8%~~ interest to produce 84?

4. What sum must be put to interest at 3% that principal and interest may amount to $590 in 6 years? At 4%, that the amount shall be $1287 in 1 yr. 6 mo.?

SLATE EXERCISES.

1. What sum of money must be invested at 4% to support 10 orphans in an asylum at a cost of $180 a year each?

2. How much must be paid for a house, that it may yield $840 per year rent at 8% on the cost?

3. Mr. Jones buys a house for $9250 and rents it for $740 a year. What per cent. does he receive on his investment?

4. For how long a time must $756 remain at interest to double itself at 6%? At 10%? At 8%?

5. At what rate per cent. must a ~~sum of~~ money be invested to double itself in 16 years?

6. What sum at 4½% interest will yield an annual income of $1800.

7. At what rate must $2300 be invested to yield an annual income of $161?

8. For what time must $2580 remain at 8% interest to amount to $3612?

9. What sum must a father invest for his daughter, now 4 years old, so that at 20 she may have $1000?

10. What sum in 6 years, at 4%, will yield the same interest that $850 will yield in 3 years at 6%?

Present Worth.

234. The present worth of a debt, payable at a future time without interest, is such a sum of money that, if put at interest, it would amount to the debt when it becomes due.

235. The difference between the face of a debt and its present worth is ***True Discount.***

True discount is so called to distinguish it from *bank discount.* True discount is the interest on the *present worth* of a debt. Bank discount is interest on the whole debt. The difference is the interest on the true discount.

Example.—Find the present worth and the true discount of a debt of $48.36, payable in 6 months without interest.

Note.—The mode of solving problems in present worth is exactly the same as that of finding the principal, the amount (or debt), time, and rate % being given.

Exact Interest.

In all the foregoing calculations, 30 days have been taken for one month, and 360 days for one year. The interest thus found is greater than if reckoned for exact time, counting 365 days to the year.

236. To obtain ***Accurate*** or ***Exact Interest*** for any number of days, we find the exact time in days, and take as many 365ths of a year's interest as there are days in the given time.

Example.—1. $350 is at interest from January 16 to March 29, 1880. What is the accurate interest?

Exact Time.		Solution.
From Jan. 16 to 31 =	15 d,	$3.50 = int. at 1 % for 1 yr.
In February	29 d.	6
To March 29	29 d.	21.00 = int. at 6 % for 1 yr.
	73 d.	$\frac{73 \times \$21}{365} = \$4.20$ = int. at 6 % for 73 d.

Find the interest as in previous cases, also the exact interest:

	Principal.	Time.	Rate.	
2.	$5000,	From Jan. 1 to Aug. 16, 1875,	6 %.	Also at 4 %.
3.	$16500,	" Feb. 10 to Sept. 15, 1879,	6 %.	" " 3 %.
4.	$24800,	" Mar. 8 to Oct. 3, 1879,	6 %.	" " 2 %.
5.	$7500,	" July 2 to Nov. 9, 1880,	5 %.	" " 7 %.

Common Business Method.

237. Bankers, and most other business men, in computing interest for short periods of time, usually take the exact number of days as above, but reckon each day as $^1/_{360}$ of a year.

This method differs from the preceding in reckoning 360 days to the year instead of 365.

In almost all the States 3 days, called ***"days of grace,"*** are allowed for the payment of a note after the promised time of payment.

Example.—1. Find the interest of $150 from April 27 to June 26, 1885, at 9%.

In April,	3 d.	1.50	Int. for 60 d. at 6%.
In May,	31 d.	75	3 " "
In June,	26 d.	2)1.575	63 " "
Grace,	3 d.	.788	
	63 d.	2.363	Int. for 63 d. at 9%.

Banking.

238. A ***Promissory Note*** is a written promise to pay a stated sum of money, either on demand or in a certain time, to some person named in the note.

239. Banks loan money on notes. For doing so, they charge interest on the note. This they deduct from the amount of the note at the time of making the loan. The remainder is called the ***avails*** or ***proceeds*** of the note.

240. A note is said to be ***payable*** at the time promised. It ***matures*** three days later (i. e., on the last day of grace). It is then ***legally due.***

241. The two dates showing when a note is *payable*, and when *legally due*, are written thus: Aug. $^5/_8$, which is read August fifth, eighth.

242. Bankers usually reckon interest according to the *Common Business Method.* (See Art. 237.)

Example.—Find the bank discount and proceeds of a note for $645, payable Sept. 20, discounted at 6% July 16.

Find the interest and proceeds of a note for

2. $450, dated Aug. 10, due Nov. 8, 1885, discounted at 6%.
3. $720, " Jan. 25, " April 7, 1885, " " 7%.
4. $960, " Feb. 3, " March 19, 1884, " " 8%.